

User guide for the hfbcs-qrpa (v1) code

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We briefly explain the structure of the Hartree-Fock-Bardeen-Cooper-Schrieffer (HFBCS) and Quasi-particle Random Phase Approximation (QRPA) code `hfbcs-qrpa`, highlighting only the differences with the code `skyrmrpa` that has been previously published [1]. The code deals with open shell spherical systems and non-spin flip, non-charge-exchange and natural parity $(-1)^l$ excitations.

I. DIFFERENCES WITH THE PUBLISHED CODE SKYRME_RPA

The present code is an extension of the Hartree-Fock plus Random Phase Approximation (HF plus RPA) code that has already been published in Ref. [1]. HF is extended to HF-Bardeen-Cooper-Schrieffer (HF-BCS) and, accordingly, RPA to Quasiparticle RPA (QRPA). The scheme remains self-consistent, once the effective pairing force is introduced.

Along the code we use $\hbar c = 197.327053$ MeV fm and $mc^2 = 938.91869$ MeV, obtained as $(m_p + m_n)/2$, for both neutrons and protons.

A. HF-BCS

HF equations are solved in coordinate space, assuming spherical symmetry, as explained in [1], and employing the standard form of a Skyrme interaction¹. Available Skyrme interactions are: SkM*, SkP, SLy4, SLy5, SkX, KDE0-J33 (KDE33) and SAMi. The HF equations are coupled to the standard BCS equations, that in spherical symmetry with $\alpha = (n, l, j)$ read

$$\sum_{\alpha} (2j_{\alpha} + 1) v_{\alpha}^2 = N, \quad (1)$$

$$\Delta_{\alpha} = - \sum_{\alpha'} \frac{\Delta_{\alpha'}}{2E_{\alpha'}} V_{\alpha\tilde{\alpha}\alpha'\tilde{\alpha}'}, \quad (2)$$

where E , u and v are the usual quasi-particle energies and BCS amplitudes as defined in the literature [3], and the tilde denotes the time-reversal state. N is the particle number. Either proton pairing, or neutron pairing, or both, can be active. No neutron-proton pairing is included and, thus, the particles retain their quantum number t_z .

To keep u and v positive, we use the phase convention in which the single-particle (s.p.) wave functions include

a factor i^l [Eq. (7) of Ref. [1] should be modified accordingly]. The number and gap equations, (1) and (2), are solved in a limited window which is specified by the user if the pairing is run in manual mode. In that case, an upper limit on the corresponding s.p. energy ε_{α} and a fixed number of maximum s.p. levels above the Fermi energy to be considered should be specified. Note that only one of those two pairing cutoffs, namely the lowest, will act as the real one. These two possibilities give more flexibility in the usage of the code. Both types of cutoffs can be different for neutrons and protons. There is no lower limit imposed for this pairing window. For those users not conversant with pairing calculations or not interested in exploring pairing effects, we strongly recommend to use pairing in automatic mode, in which case, a volume pairing force (see below) has been fitted to reproduce the experimental neutron pairing gap in ^{120}Sn ($\Delta = 1.245$ MeV) with a neutron pairing window fixed by taking into account only bound states above the neutron Fermi energy and to a maximum of 6 neutron s.p. levels. The default pairing window in automatic mode is set in an analogous way.

The matrix elements $V_{\alpha\tilde{\alpha}\alpha'\tilde{\alpha}'}$ are calculated using a zero-range, density-dependent pairing force of the type

$$\begin{aligned} V(\mathbf{r}_1, \mathbf{r}_2) &= V_0 \left[1 + x \left(\frac{\rho(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2})}{\rho_0} \right) \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \\ &\equiv V_{\text{pair}}(r) \delta(\mathbf{r}_1 - \mathbf{r}_2). \end{aligned} \quad (3)$$

In the above equation, $\rho_0 = 0.16 \text{ fm}^{-3}$ while the other two parameters x and V_0 have to be adjusted. $x = 0, 1$ and 0.5 define the usual volume, surface and mixed pairing. V_0 is usually adjusted to reproduce some given pairing gap but it has to be noted that the code does not make this adjustment by itself unless automatic mode is chosen where only the use of volume pairing ($x = 0$) is allowed.

The total HF-BCS energy can be calculated, and is calculated by the code, in two different manners. It can be calculated from the force, or energy functional, directly. In this case

$$E = E_{\text{kin}} + E_{\text{Skyrme}} + E_{\text{Coul}} + E_{\text{pair}}. \quad (4)$$

The first two terms are the kinetic and Skyrme contributions to the energy and are written as the volume integrals of the corresponding energy densities, $\mathcal{E}_{\text{kin}}$ and

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¹ In the current version, tensor terms [2] are implemented at the HF-BCS level but not in the residual QRPA interaction.

$\mathcal{E}_{\text{Skyrme}}$, as in Eqs. (2.1) and (2.2) of Ref. [4], or in Eqs. (24) and (38) of Ref. [5]. The Coulomb energy has a direct part and an exchange part that is calculated in the usual Slater approximation. The pairing energy is written as in Eq. (16) of [5]. As an alternative,

$$E = \frac{1}{2} \sum_{\alpha} v_{\alpha}^2 \varepsilon_{\alpha} + \frac{1}{2} E_{\text{kin}} + E_{\text{rearr}} + E_{\text{pair}}, \quad (5)$$

where the first term includes the s.p. energies ε_{α} , and E_{rearr} is the rearrangement term associated with the density-dependent term in the Skyrme force (as in Eq. (109) of [5]), as well as with the Coulomb exchange energy within the Slater approximation. The equality between the two expressions (4) and (5) for the total energy is, among the rest, a useful test for the convergence of the HF-BCS equations.

Regarding the calculation of charge radii, we take into account the electromagnetic finite size of the proton and the neutron. That is, we assume the proton and neutron electric sizes as $\langle r_p^2 \rangle = 0.64 \text{ fm}^2$ and $\langle r_n^2 \rangle = -0.11 \text{ fm}^2$, respectively, as well as the anomalous proton magnetic moment $\kappa_p = \mu_p - 1$ with $\mu_p = 2.79$ and neutron magnetic moment $\kappa_n = \mu_n = -1.91$. The expression for the charge radius $\langle r^2 \rangle_{\text{ch}}$ used is derived in the non-relativistic limit and can be written as [6] (see also [7]),

$$\begin{aligned} \langle r^2 \rangle_{\text{ch}} &= \langle r^2 \rangle_{\text{p}} + \langle r^2 \rangle_{\text{n}} + \frac{N}{Z} \langle r_n^2 \rangle \\ &+ \frac{1}{Z} \left(\frac{\hbar}{mc} \right)^2 \sum_{\alpha\tau} v_{\alpha\tau}^2 (2j_{\alpha} + 1) \kappa_{\tau} \langle \boldsymbol{\sigma} \cdot \mathbf{l} \rangle_{\alpha}, \quad (6) \end{aligned}$$

where $\tau = p, n$ indicates if the nucleon is a proton or a neutron, respectively, and $\langle r^2 \rangle_{\text{p}}$ is the mean square radius

of the density distribution of protons,

$$\langle r^2 \rangle_{\text{p}} = \int d\mathbf{r} r^2 \rho_p(\mathbf{r}). \quad (7)$$

We also evaluate the isospin mixing ε in the obtained s.p. wave-functions as discussed in Sec. IIA of Ref. [8]. Specifically,

$$\varepsilon^2 \equiv \frac{1}{N - Z + 2} \sum_{\substack{n_p, n_n \\ l_p = l_n \\ j_p = j_n}} (2j_p + 1) v_p^2 u_n^2 \mathcal{O}_{np}^2 \quad (8)$$

where we have omitted the α label (previously defined) for the sake of simplicity and clarity. The overlap factor $\mathcal{O}_{np}$ between the neutron (n) and proton (p) radial part $R_{n,l}(r)$ of the considered s.p. wave function is defined as

$$\mathcal{O}_{np} \equiv \int_0^{\infty} dr r^2 R_{n_p, l_p}(r) R_{n_n, l_n}(r). \quad (9)$$

B. QRPA

The QRPA equations are

$$\begin{pmatrix} A_{\alpha\beta, \gamma\delta} & B_{\alpha\beta, \gamma\delta} \\ -B_{\alpha\beta, \gamma\delta}^* & -A_{\alpha\beta, \gamma\delta}^* \end{pmatrix} \begin{pmatrix} X_{\gamma\delta}^{\nu} \\ Y_{\gamma\delta}^{\nu} \end{pmatrix} = E_{\nu} \begin{pmatrix} X_{\alpha\beta}^{\nu} \\ Y_{\alpha\beta}^{\nu} \end{pmatrix}. \quad (10)$$

The basis on which these equations are written is made up with two quasiparticle states, like $(\alpha\beta)$ and $(\gamma\delta)$. These respect the usual selection rules in that they are coupled to good angular momentum J and parity π . The upper limit on the two quasiparticle energy can be specified (see below). On the other hand, the code automatically discards configurations for which both quasiparticle states are fully occupied or both have an occupation probability lower than 10^{-6} .

The matrix elements A and B , in the angular momentum-coupled representation, written on the HF-BCS basis two-quasiparticle basis, have the form

$$\begin{aligned} A_{\alpha\beta, \gamma\delta} &= \frac{1}{\sqrt{1 + \delta_{\alpha\beta}} \sqrt{1 + \delta_{\gamma\delta}}} \\ &\times [(E_{\alpha} + E_{\beta}) \delta_{\alpha\gamma} \delta_{\beta\delta} + G_{\alpha\beta\gamma\delta} (u_{\alpha} u_{\beta} u_{\gamma} u_{\delta} + v_{\alpha} v_{\beta} v_{\gamma} v_{\delta}) \\ &+ F_{\alpha\beta\gamma\delta} (u_{\alpha} v_{\beta} u_{\gamma} v_{\delta} + v_{\alpha} u_{\beta} v_{\gamma} u_{\delta}) - (-1)^{j_{\gamma} + j_{\delta} - J'} F_{\alpha\beta\delta\gamma} (u_{\alpha} v_{\beta} v_{\gamma} u_{\delta} + v_{\alpha} u_{\beta} u_{\gamma} v_{\delta})], \quad (11) \end{aligned}$$

$$\begin{aligned} B_{\alpha\beta, \gamma\delta} &= \frac{1}{\sqrt{1 + \delta_{\alpha\beta}} \sqrt{1 + \delta_{\gamma\delta}}} \\ &\times \left[-G_{\alpha\beta\delta\gamma} (u_{\alpha} u_{\beta} v_{\gamma} v_{\delta} + v_{\alpha} v_{\beta} u_{\gamma} u_{\delta}) - (-1)^{j_{\delta} + j_{\gamma} - J'} F_{\alpha\beta\delta\gamma} (u_{\alpha} v_{\beta} u_{\gamma} v_{\delta} + v_{\alpha} u_{\beta} v_{\gamma} u_{\delta}) \right. \\ &\left. + (-1)^{j_{\alpha} + j_{\beta} + j_{\gamma} + j_{\delta} - J - J'} F_{\alpha\beta\gamma\delta} (u_{\alpha} v_{\beta} v_{\gamma} u_{\delta} + v_{\alpha} u_{\beta} u_{\gamma} v_{\delta}) \right], \quad (12) \end{aligned}$$

with

$$G_{\alpha\beta\gamma\delta} = \sum_{m_\alpha m_\beta m_\gamma m_\delta} \langle j_\alpha m_\alpha j_\beta m_\beta | JM \rangle \langle j_\gamma m_\gamma j_\delta m_\delta | J' M' \rangle V_{\alpha\beta,\gamma\delta}^{pp}, \quad (13)$$

$$F_{\alpha\beta\gamma\delta} = \sum_{m_\alpha m_\beta m_\gamma m_\delta} \langle j_\alpha m_\alpha j_\beta m_\beta | JM \rangle \langle j_\gamma m_\gamma j_\delta m_\delta | J' M' \rangle V_{\alpha\delta\beta\gamma}^{ph}. \quad (14)$$

$V_{\alpha\delta\beta\gamma}^{ph}$ are the (uncoupled) matrix elements of the particle-hole effective interaction, and $V_{\alpha\beta,\gamma\delta}^{pp}$ represent the (uncoupled) matrix elements of the particle-particle effective interaction. The p-h effective interaction is de-

scribed in detail in Ref. [1]. In fact, one can immediately spot that the matrix element in Eq. (14) is the same that has been defined in Eq. (16) of Ref. [1]; the different notation here has the purpose of distinguishing it clearly from the matrix element G that is the one of the pairing force and reads

$$G_{\alpha\beta\gamma\delta} = R \sum_{\lambda} \left((-)^{j_\beta+j_\gamma+J} \left\{ \begin{matrix} j_\alpha & j_\gamma & \lambda \\ j_\delta & j_\beta & J \end{matrix} \right\} \langle \alpha || Y_\lambda || \gamma \rangle \langle \beta || Y_\lambda || \delta \rangle + (-)^{j_\beta+j_\gamma} \left\{ \begin{matrix} j_\alpha & j_\delta & \lambda \\ j_\gamma & j_\beta & J \end{matrix} \right\} \langle \alpha || Y_\lambda || \delta \rangle \langle \beta || Y_\lambda || \gamma \rangle \right), \quad (15)$$

where R is the radial integral

$$R = \int \frac{dr}{r^2} u_\alpha(r) u_\beta(r) u_\gamma(r) u_\delta(r) V_{\text{pair}}(r). \quad (16)$$

The excitation operators and associated sum rules used in the code are those discussed in Ref. [1]. Regarding spurious states: the HF-BCS is known to break different symmetries present in the original Hamiltonian. In principle QRPA should restore those symmetries exactly. However, numerical implementations are not exact, and

spurious states do not appear necessarily at zero energy as expected. As a rule, the spurious state should be the first excited state, close to zero energy, in the calculated spectrum. Specifically, within the present code: i) for 0^+ excitations, we have not implemented any projection technique to avoid the spurious state originated from the violation of the particle number; and ii) for 1^- excitations, we use the operators defined in [1] to approximately remove the spurious state that arise from the violation of translational invariance.

II. INPUT OF THE CODE

In this section we briefly describe the input file `hfbcscs-qrpa.in` needed to run the code. The format of the input file should be as follows:

Line	Input	Explanation
1	FORCE	Name of the Skyrme interaction to be used [options: SkM*, SkP, SLy4, SLy5, SkX, KDE33, SAMi]
2	MESH STEP	Number of mesh points and step (fm)
3	A Z	Mass number (A) and proton number (Z)
4	EPS	Required accuracy for the HFBCS equations. Specifically, it corresponds to the maximum accepted difference between consecutive iterations and it is used for: single-particle energy, gap and chemical potential.
5	FLAG BCSP BCSN	Automatic (0) or manual(1) BCS calculations. Active (1) non-active (0) for protons. Active (1) non-active (0) for neutrons. Only if manual mode has been chosen next three lines (6-8) should be added:
6	LEVP LEVN	Number of levels taken above the Fermi level in the BCS calculation for protons and neutrons.
7	EMAXP EMAXN	Maximum proton and neutron energies for the BCS calculation.
8	-V_0 x	Parameters of the pairing interaction to be used defined in the previous section.
9	CALEX	Type of calculations [options:

```

- UNP: stands for unperturbed calculation, residual interaction set to zero
- TDA: Tamm-Dancoff Approximation is used
- RPA: Random Phase Approximation is used].
10  J P      Total angular momentum and parity (+1 or -1). At the moment only natural
parity is considered.
11  ECP ECN   Minimum and maximum energy for the considered two body
neutron-neutron and proton-proton basis states.

```

III. OUTPUT OF THE CODE

The code produces different output files. Definitions of the different quantities such as operators or transition densities are as in SKYRME_RPA [1]. Below a brief description:

File name	Description
-----	-----
hfbcqs-qrpa.out	main output of the code
quasiparticle_states.out	full list of quasi-particle states used in the calculation. Information on the Fermi energies is also given
basis_states.out	full list of two-body neutron-neutron and proton-proton basis states used in the calculation.
d.out	proton and neutron densities
td.out	proton and neutron transition densities for all excited states
strength_*.out	three files contain isoscalar, isovector and electromagnetic strength functions (reduced transition probabilities convoluted by a Lorentzian function with a energy step of 0.1 MeV and width of 1 MeV)

In the main output file (**hfbcqs-qrpa.out**) details on the parameters of the used functional as well as active terms are printed. One-body center of mass correction is active for all the Skyrme interaction assuming the same prescription as in SKYRME_RPA. Convergence information and a list of quasi-particle levels is given. Total binding energy as well as its different contributions are printed in two different forms (see the discussion above). Firstly, the energy is calculated as the integral of the energy density. Then, the energy is calculated via the sum of the eigenvalues. In this latter case the rearrangement energy is needed. The calculation of the binding energy in two different ways serves as a strong test for the convergence the code. Neutron and proton root mean square radii are also printed and the root mean square radius is estimated. Average pairing gaps, and isospin mixing in the ground state wave function are also given in the output. Finally, different sum rules and isoscalar, isovector and electromagnetic reduced transition probabilities are listed.

IV. EXAMPLE

In this section, we show an input (**hfbcqs-qrpa.in**) example of the code which uses the manual pairing option (FLAG=1):

```

SLy5
200 0.1
120 50
1.d-6
1 0 1
0 6
0. 0.
870.6 1.
RPA
2 +1
0. 100.

```

The main output (**hfbcqs-qrpa.out**) of the code corresponding to this input is given below.

```
-Error spe      0.10E-05
-Error gap eq.  0.10E-05
-Error num. eq. 0.10E-05
```

POTENTIAL INCLUDED IN THE CALCULATIONS:

```

-SKYRME STANDARD WITH
      |___ J2 TERMS... YES
      |___ TENSOR..... NO
-COULOMB DIRECT..... YES
-COULOMB EXCHANGE..... YES
-SPIN-ORBIT..... YES
-DEFAULT PAIRING..... NO
-PROT. PAIRING (BCS).. NO
-NEUT. PAIRING (BCS).. YES
      ENERGY CUT..... 0.0000000000000000E+000
      LEV. ABOVE F.... 6
Type: delta force
Vp= V0(1-x(rho/rho0))^g
V0 = 0.87060E+03
x = 0.10000E+01
rho0= 0.16000E+00
g = 0.10000E+01
-CENTER OF MASS CORR.. YES

```

CONVERGENCE:

```
-At iteration number:          77
-Maximum error in spe: 0.99830E-05
```

[illegible]

PROTON STATES

1S1/2	E=-0.4664E+02	DEG= 0.2000E+01	V^2= 0.1000E+01	D= 0.0000
1P3/2	E=-0.3873E+02	DEG= 0.4000E+01	V^2= 0.1000E+01	D= 0.0000
1P1/2	E=-0.3778E+02	DEG= 0.2000E+01	V^2= 0.1000E+01	D= 0.0000
1D5/2	E=-0.2980E+02	DEG= 0.6000E+01	V^2= 0.1000E+01	D= 0.0000
1D3/2	E=-0.2756E+02	DEG= 0.4000E+01	V^2= 0.1000E+01	D= 0.0000
2S1/2	E=-0.2559E+02	DEG= 0.2000E+01	V^2= 0.1000E+01	D= 0.0000
1F7/2	E=-0.2040E+02	DEG= 0.8000E+01	V^2= 0.1000E+01	D= 0.0000
1F5/2	E=-0.1634E+02	DEG= 0.6000E+01	V^2= 0.1000E+01	D= 0.0000
2P3/2	E=-0.1522E+02	DEG= 0.4000E+01	V^2= 0.1000E+01	D= 0.0000
2P1/2	E=-0.1361E+02	DEG= 0.2000E+01	V^2= 0.1000E+01	D= 0.0000
1G9/2	E=-0.1083E+02	DEG= 0.1000E+02	V^2= 0.1000E+01	D= 0.0000

NEUTRON STATES

1S1/2	E=-0.5616E+02	DEG= 0.2000E+01	V^2= 0.1000E+01	D= 0.2017
1P3/2	E=-0.4748E+02	DEG= 0.4000E+01	V^2= 0.1000E+01	D= 0.4678
1P1/2	E=-0.4615E+02	DEG= 0.2000E+01	V^2= 0.1000E+01	D= 0.4191
1D5/2	E=-0.3787E+02	DEG= 0.5999E+01	V^2= 0.9998E+00	D= 0.7963
1D3/2	E=-0.3481E+02	DEG= 0.3999E+01	V^2= 0.9998E+00	D= 0.7045
2S1/2	E=-0.3334E+02	DEG= 0.1999E+01	V^2= 0.9997E+00	D= 0.8519
1F7/2	E=-0.2775E+02	DEG= 0.7993E+01	V^2= 0.9991E+00	D= 1.1555
1F5/2	E=-0.2250E+02	DEG= 0.5992E+01	V^2= 0.9987E+00	D= 1.0469
2P3/2	E=-0.2194E+02	DEG= 0.3993E+01	V^2= 0.9982E+00	D= 1.1819
2P1/2	E=-0.2013E+02	DEG= 0.1995E+01	V^2= 0.9975E+00	D= 1.1866
1G9/2	E=-0.1747E+02	DEG= 0.9937E+01	V^2= 0.9937E+00	D= 1.4813
2D5/2	E=-0.1137E+02	DEG= 0.5787E+01	V^2= 0.9644E+00	D= 1.2543
1G7/2	E=-0.0976E+01	DEG= 0.7133E+01	V^2= 0.8916E+00	D= 1.3924
3S1/2	E=-0.9077E+01	DEG= 0.1618E+01	V^2= 0.8089E+00	D= 1.0877

2D3/2	E=-0.8573E+01	DEG= 0.2530E+01	V^2= 0.6325E+00	D= 1.2771
1H11/2	E=-0.7227E+01	DEG= 0.2978E+01	V^2= 0.2481E+00	D= 1.7057
2F7/2	E=-0.1526E+01	DEG= 0.4533E-01	V^2= 0.5666E-02	D= 1.0167
3P3/2	E=-0.1663E-01	DEG= 0.2758E-02	V^2= 0.6894E-03	D= 0.4313
2F5/2	E= 0.1169E+01	DEG= 0.0000E+00	V^2= 0.0000E+00	D= 0.0000
1H9/2	E= 0.1856E+01	DEG= 0.0000E+00	V^2= 0.0000E+00	D= 0.0000
1I13/2	E= 0.2809E+01	DEG= 0.0000E+00	V^2= 0.0000E+00	D= 0.0000

[illegible]

CONTRIBUTIONS TO THE TOTAL ENERGY (MeV):

[illegible]

-INTEGRAL OF THE ENERGY DENSITY:

```

E(KIN) = 0.21769E+04
E(SKYRME) = -0.34793E+04
|___E(t0) = -0.12698E+05
|___E(t3) = 0.82849E+04
|___E(t1,t2) = 0.93408E+03
E(SO) = -0.54885E+02
E(CD) = 0.36706E+03
E(CE) = -0.19131E+02
E(pair) = -0.99067E+01
E(TOT) = -0.10193E+04

```

- HF ENERGY + E_REA:

```

E(KIN) =      0.10885E+04
E(SPE) =     -0.14010E+04
E(REA) =     -0.69678E+03
E(pair) =     -0.99067E+01
E(TOT) =     -0.10193E+04

```

-NUMERICAL CHECKS:

E_INT/E_HF-1
REL. ERR. (%) = 0.36779E-03

[illegible]

ROOT MEAN SQUARE RADII

[illegible]

R(N) =	0.47278E+01
R(P) =	0.45865E+01
R(CH) =	0.46418E+01
R ² (CH) =	0.21546E+02
___R ² (p) =	0.64000E+00
___N*R ² (n)/Z=	-0.15400E+00
___R ² (S0-n) =	-0.38991E-01
___R ² (S0-p) =	0.63250E-01

[illegible]

AVERAGE PAIRING GAPS (MeV):

[illegible]

```
GAP(P) = 0.00000E+00
GAP(n) = 0.14505E+01
```

[illegible]

ISOSPIN MIXING IN THE HF G.S.


```

4 0.36154E+01 0.59934E+03 0.22688E+01 0.63858E+02 0.72237E+02 0.64195E+00
5 0.43815E+01 0.33156E+02 0.12551E+00 0.96100E+01 0.19523E+00 0.17350E-02
6 0.46192E+01 0.19115E+02 0.72361E-01 0.62297E+01 0.38413E+00 0.34136E-02
7 0.48144E+01 0.68087E+01 0.25774E-01 0.13742E+01 0.70107E-01 0.62302E-03
8 0.56035E+01 0.10373E+04 0.39268E+01 0.48223E+03 0.13716E+03 0.12189E+01
9 0.56320E+01 0.37730E+03 0.14283E+01 0.18057E+03 0.55514E+02 0.49333E+00
10 0.66789E+01 0.11180E+02 0.42321E-01 0.14908E+00 0.66120E+01 0.58758E-01
. . .
1080 0.98378E+02 0.28779E-05 0.10894E-07 0.14364E-02 0.60056E-02 0.53369E-04
1081 0.98572E+02 0.27497E-04 0.10409E-06 0.21067E-02 0.74915E-02 0.66575E-04
1082 0.98620E+02 0.10907E-04 0.41290E-07 0.41446E-04 0.26174E-03 0.23260E-05
1083 0.99015E+02 0.52814E-04 0.19993E-06 0.38653E-04 0.26697E-04 0.23725E-06
1084 0.99206E+02 0.33656E-03 0.12741E-05 0.80204E-02 0.25846E-01 0.22969E-03
1085 0.99424E+02 0.72734E-03 0.27533E-05 0.35527E-04 0.22645E-03 0.20124E-05
1086 0.99529E+02 0.44898E-01 0.16996E-03 0.14265E-04 0.48156E-01 0.42795E-03
1087 0.99840E+02 0.12720E-04 0.48151E-07 0.12853E-02 0.46425E-02 0.41256E-04
1088 0.99994E+02 0.68463E-04 0.25917E-06 0.19422E-02 0.92958E-02 0.82608E-04
1089 0.10014E+03 0.25707E-02 0.97315E-05 0.10805E-02 0.22619E-03 0.20101E-05
1090 0.10151E+03 0.25770E-02 0.97551E-05 0.44132E-04 0.14046E-02 0.12482E-04

```

CPU TIME IN SECONDS = 2017.41

Finally, for this example, we show in Fig. 1 the reduced transition probabilities contained in `hfbcs-qrpa.out` and their convolution with a Lorentzian function with a width of 1 MeV. The latter information is also given by the code and can be found in the file `strength.is.out` for the isoscalar response and `strength.iv.out` for the isovector response.

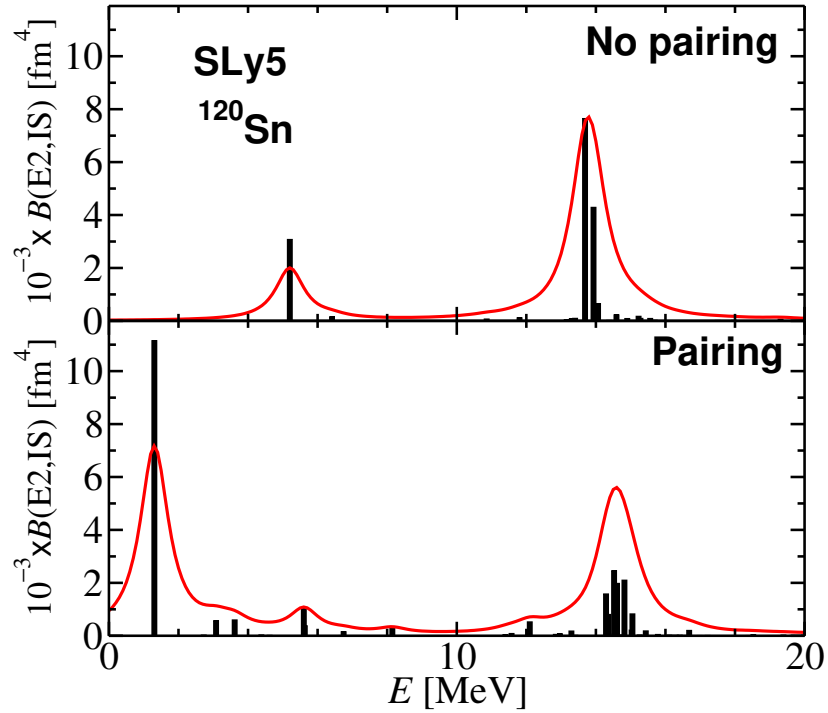


FIG. 1. Reduced transition probabilities (bars) for the text example contained in `hfbcs-qrpa.out` and their convolution with a Lorentzian function (solid lines) with a width of 1 MeV. The latter information is also given by the code and can be found in the file `strength.is.out` for the isoscalar response (upper panel) and `strength.iv.out` for the isovector response (lower panel).

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