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Electron scattering as a probe of surface properties and proton shell structure of unstable nuclei

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Theoretical study of elastic electron scattering off stable and exotic nuclei **X. Roca-Maza, M. Centelles, F. Salvat, and X. Viñas Phys. Rev. C 78, 044332 (2008).**

Electron scattering in isotonic chains as a probe of the proton shell structure of unstable nuclei **X. Roca-Maza, M. Centelles, F. Salvat, and X. Viñas Phys. Rev. C 87, 014304 (2013).**

March 8, 2013
RIKEN Nishina Center, RIBF Nuclear Physics Seminar

Motivation

- **In-medium nuclear (effective) interaction is not well understood for extreme values of isospin asymmetry, that is, far from the stability valley**
- **Experimental studies of elastic electron scattering by unstable nuclei:**
  - will be **feasible** in rare ion beam facilities
  - determine e-m charge distribution **model independently**
  - better understanding of nuclei under more **extreme conditions**
  - data on **large isospin asymmetries**
- **Theoretical studies of elastic electron scattering by unstable nuclei:**
  - Physical process **well understood** since many years ago.
  - **Exact calculations available** once the exact electromagnetic charge distribution is known
  - **Theoretical guidance for future experiments**

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Exotic Nuclei: New experimental landscape
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- Results & Correlations
- Conclusions

Exact solution: Dirac partial-wave (also known as DWBA) calculation of elastic scattering of electrons by nuclei. X. Roca-Maza *et al.* Phys. Rev. C **78**, 044332 (2008) & F. Salvat *et al.* Comp. Phys. Comm. **165** 157-190 (2005)

For studies on the nuclear charge distribution:

- $\mathbf{E}_{\rm beam} \sim 2\pi \frac{\hbar c}{\lambda_{\rm Nucl.size}}$ where $\lambda_{\rm nucl.size} \sim 2\langle r^2\rangle^{1/2} \sim 2 - 10$ fm $\Rightarrow$ **100 − 600 MeV**.
- **Relativistic treatment** is needed $m_e c^2/E_{\rm beam} \lesssim 0.005$.
- At these energies, effect of **screening by the orbiting atomic electrons** is limited to scattering angles **smaller than 1 degree** (we will not calculate them here).
- The interaction potential is $V_{\rm nucl.elec.}$ calculated from $\rho_{\rm ch}$ (parametrized, model, ...)

$$V_{\rm nucl.elec.} = 4\pi Z_0 e^2 \left\{ \frac{1}{r}\int_0^r \rho_{\rm ch}(u)u^2 du + \int_r^\infty \rho_{\rm ch}(u)udu \right\}$$

- spherical symmetry assumed

Differential cross section for unpolarized electrons:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |g(\theta)|^2$$

where f and g are the **direct and spin-flip amplitudes** determined from the solution of the **Dirac equation for a central potential** $V_{\mathrm{nucl.elec.}}(r)$ (asymptotically as a plane wave + outgoing spherical wave)

f and g admit a **partial-wave expansion** in terms of,

- the **phase-shifts** that represents the large r behavior of **Dirac spherical waves** $\Psi(\mathbf{r})$ function of the spherical spinors and two radial functions.
- Legendre and associated **Legendre polynomials**.

Sherman function: degree polarization from initially unpolarized beam scattered at angle θ

$$S(\theta) \equiv i\,\frac{f(\theta)g^*(\theta) - f^*(\theta)g(\theta)}{|f(\theta)|^2 + |g(\theta)|^2}$$

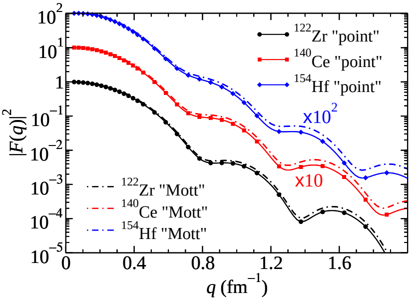
It may help in setting up detectors

Form factor

$$|F_{DWBA}(q)|^2 \equiv \frac{d\sigma/d\Omega}{d\sigma_{\rm point}/d\Omega}$$

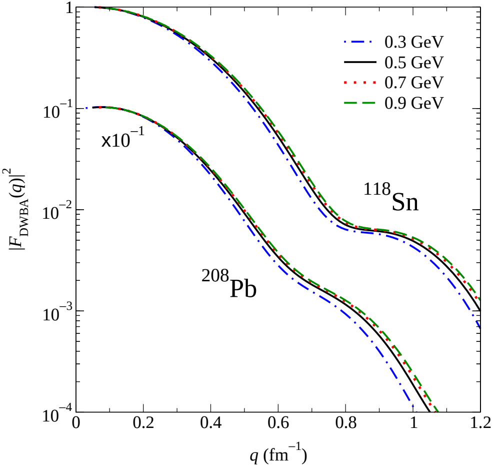
where $d\sigma_{\rm point}/d\Omega$ is the DWBA solution for a point nucleus and $c\hbar q = 2E\sin(\theta/2)$.

- This definition, as compared to $\frac{d\sigma/d\Omega}{d\sigma_{\rm Mott}/d\Omega}$, disentagles better the finite size effects of the nucleus.
- Nevertheless, it is found that the choice is not critical for our study in the low momentum transfer regime.



Mott DCS: $\frac{d\sigma_{\rm Mott}}{d\Omega} = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\theta}{\sin^4\theta}$; for small angles diverges as θ^{-4}

Energy Dependence in the e-m Form Factor

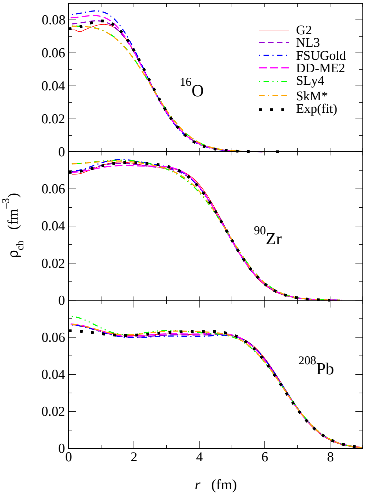


Test: The form factor in DWBA is almost energy-independent in the region we are interested in (low q-regime).

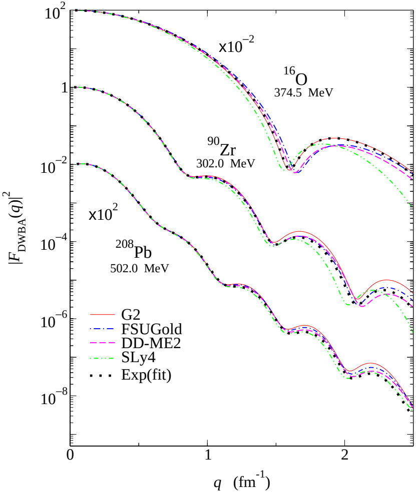
$F_{DWBA}(q)$ **is a good quantity for the study of the electromagnetic structure of the nucleus**

Experiment versus Theory in stable nuclei

Nuclear Model (NM) provides:

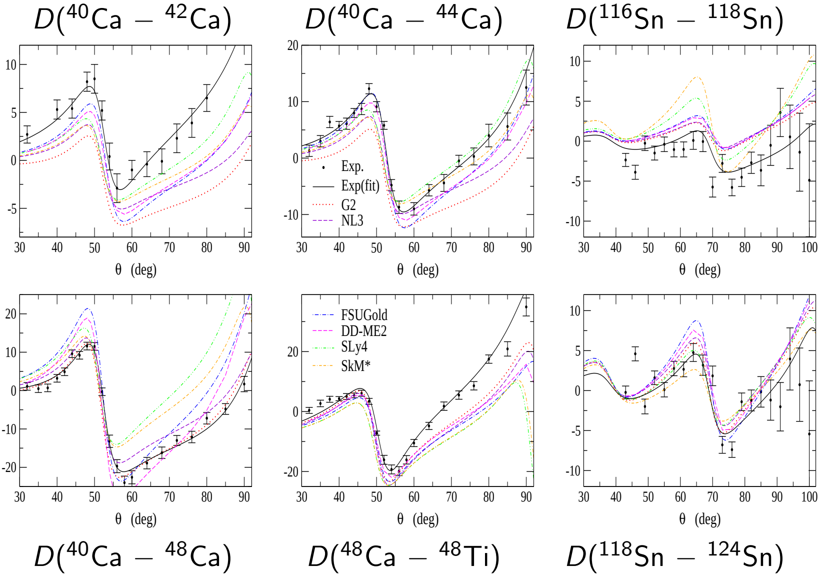


NM+DWBA provides:



… and a more demanding test:

$D(A - B) \equiv (A - B)/(A + B)$



How reliable are modern Energy Density Functionals?

In $F(q)$ and $d\sigma(\theta)$:

- Exist a quasi-model-independent q-regime: **Up to $1 - 1.5$ fm^{-1} for the studied models and scattering processes.**
- Exist discrepancies between models in the **high q-regime** **(remember differences in $\rho_{\rm ch}$ at low r)**
- If we define a **distance**, namely d_w, between experimental and theorical results:

$$d_w^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{d\sigma_i^{exp} - d\sigma_i^{theo}}{\delta\sigma_i^{exp}}\right)^2$$

  we observe that the criteria does not decide between models

Ability test for the models

Nucleus	E_e MeV	d_w^2 Exp. fit	DD-ME2	G2	NL3	FSUGold	SLy4	SKM*	"Best" model
^{16}O	374.5	**11.1**[b]	88.7	13.1	38.6	206.	191.	194.	**G2**
^{40}Ca	250.0	**7.18**[b]	3.15	16.2	13.9	0.84	24.4	24.3	**FSUGold**
	500.0	**3.48**[b]	1.49	42.9	19.7	5.79	40.0	39.0	**DD-ME2**
^{48}Ca	250.0	**6.66**[b]	4.85	9.74	7.14	4.08	14.9	13.6	**FSUGold**
	500.0	**3.19**[b]	1.11	17.0	3.53	2.57	21.84	18.5	**DD-ME2**
^{90}Zr	209.6	**0.78**[b]	0.87	2.21	1.36	0.65	6.53	5.36	**FSUGold**
	302.0	**0.86**[b]	0.91	9.92	3.27	0.67	9.35	7.19	**FSUGold**
^{118}Sn	225.0	**5.43**[a]	18.4	34.8	25.5	31.8	2.75	4.20	**SLy4**
^{208}Pb	248.2	**30.6**[b]	44.4	154.	74.8	89.5	89.2	61.0	**DD-ME2**
	502.0	**21.2**[b]	14.1	186.	50.5	61.1	95.9	76.5	**DD-ME2**
$D(^{40}Ca - ^{42}Ca)$	250.0	**0.56**[c]	9.1	28.3	16.0	11.1	9.1	12.9	**DD-ME2**/**SLy4**
$D(^{40}Ca - ^{44}Ca)$	250.0	**1.14**[c]	4.5	29.6	12.2	3.88	7.08	9.13	**FSUGold**
$D(^{40}Ca - ^{48}Ca)$	250.0	**1.06**[c]	16.4	4.89	7.74	38.5	94.1	49.3	**G2**
$D(^{48}Ca - ^{48}Ti)$	250.0	**2.49**[c]	18.0	19.6	31.0	37.8	71.8	64.9	**DD-ME2**
$D(^{116}Sn - ^{118}Sn)$	225.0	**2.05**[a]	8.05	7.80	9.00	10.1	13.2	18.5	**G2**
$D(^{118}Sn - ^{124}Sn)$	225.0	**4.03**[a]	5.35	6.98	7.50	9.22	7.05	7.18	**DD-ME2**

[a] A.S. Litvinenko et al., Nucl.Phys. **A182**, 265 (1972). [b] B. Dreher, J. Friedrich, K. Merle, H. Rothhaas and G. Luhrs, Nucl.Phys.**A235**, 219 (1974). [c] R.F. Frosch et al. Phys. Rev. **174** (1968) 1380.

Conclusions

- Theory is well understood and calculations are feasible: **exact solution of the scattering process** once the nuclear e-m charge distribution has been provided.
- **disagreement** with the experiment **due exclusively** to the **nuclear model**
- The defined **Form Factor**
  - include **all finite size effects**
  - is **nearly energy-independent** at low momentum transfer
- d_w^2 does not decide between the different models

New experimental landscape:

e - Rare Isotope Beams

Image from the international conference *EURORIB'08*@Giens.

ELISe@FAIR and SCRIT@RIKEN projects will involve elastic electron scattering

Self-Confinig Rare Isotope Target (e-RI scattering) - SCRIT
ELectron-Ion Scattering in a Storage Ring (eA collider) - ELISe

"... exotic nuclei on the neutron rich side attract most attention in today's nuclear physics... "

"1.1.3.2 Experiments with stored beams:
...elastic and inelastic electron scattering with stored and cooled exotic nuclei."

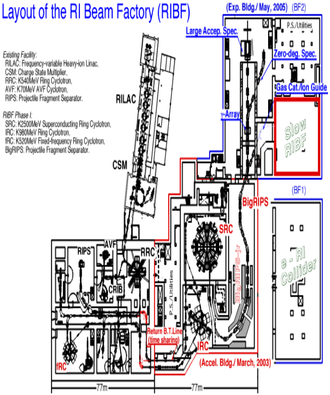


Figure taken from http://www.riken.jp/ap/conf/slowri/ and text from http://www.gsi.de/GSI-Future/cdr/ Sec. 2 Page 80 and 109 and Sec. 3 Page 489.

How can theory help in the experimental analysis? Could we find simple and general trends for $F(q)$ in exotic nuclei?

Proposed methodology:

1.- predict $F_{\mathrm{DWBA}}(q)$ along isotopic and/or isotonic chains, **from the neutron-rich to proton-rich side**

2.- use **NM + DWBA** results within the low q-region **as a guide**

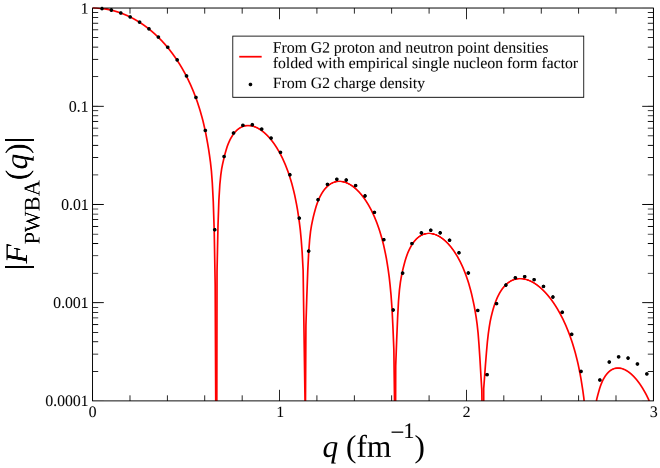
3.- parameterize the charge form factor within the **Helm model**

$\Rightarrow$ insights on the evolution of the surface and bulk contibutions to the charge distribution as the isospin number increases

- **Advantages**: Helm model has two parameters with clear physical meaning and analytic expression of $F(q)$
- **Disadvantages**: Helm model assumes PWBA, it is (in principle) needed to introduce a $q_{effective}$ that corrects the e-m effects on the phase of the incident electron wave functions.

4.- as a **benchmark NM** we will use **G2 parameter set** because it takes into account explicitly the electromagnetic structure of the nucleon self-consistently within the fitting procedure

Is the electromagnetic structure of the nucleon properly introduced within the G2 model?



Helm Model: **2** parameters

- Helm Charge Form Factor: R_0 & σ

$$F_H(q) = \int e^{i\vec{q}\vec{r}} \rho_H(\vec{r}) d\vec{r} = \frac{3}{\mathbf{R_0} q} j_1(q\mathbf{R_0}) e^{-\sigma^2 q^2/2}$$

  where σ measures the surface fall-off of the density distribution and R_0 measures its bulk extension.

- How we determine the parameters:
  - R_0**:** one requires that the first zero of F_H occurs at the same q of F_{PWBA} (fourier transform of the self-consistent density). Therefore, it coincides with the sharp radius.
  - σ**:** is chosen to reproduce the height of the second maximum of $|F_{PWBA}|$

$q_{\rm eff}$: **1** parameters

- **Effect** of the nuclear electrostatic **potential** on the **momentum transfer** (classical estimation):

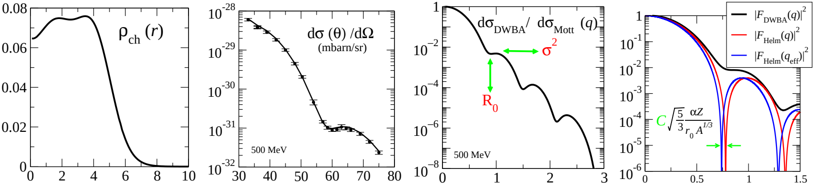
  $$\langle \hbar c q_{\rm eff}\rangle \approx \langle \hbar c q - V\rangle = \int \left(\hbar c q - V(r)\right)\rho_{\rm ch}(r)d\mathbf{r}$$

  considering a constant charge distribution in a spherical nucleus,

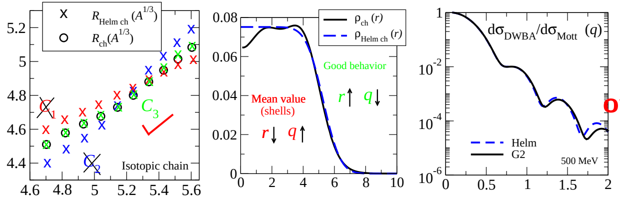
  $$q_{eff} = q\left(1 + \mathbf{C}\frac{Z\alpha}{qR_{ch}}\right) \xrightarrow{m_e \ll q_e} \quad q\left(1 + \mathbf{C}\frac{Z\hbar c\alpha}{ER_{ch}}\right)$$

  where $R_{ch} = \sqrt{3/5}r_0 A^{1/3}$ and C=6/5
- One can always **reproduce** approximately the neglected **Coulomb phase shifts fitting** C in order to **reproduce** the **measured minima in** $F(q)$.
- **Isotopic chains:** correction produce a **change in the momentum transfer** of the **same order** than the change **produced** by adding/subtracting two more **neutrons**.
- **Isotonic chains:** correction can be **neglected** (for our purposes here) since the addition/substraction of two more **protons impact much more the momentum transfer**.

Method Scheme:



$+$ DWBA $\rightarrow$ $d\sigma/d\Omega$ $\rightarrow$ $F_{DWBA}(q)$ $\rightarrow$Helm Model $+$ $q_{\rm eff}$(**C**) $\rightarrow$



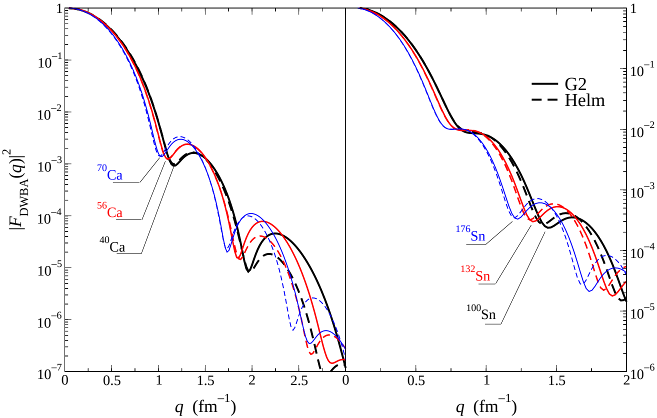
Good method within the overlapped region

C | best $R_{\rm ch}$ $+$ DWBA $\rightarrow$ **cross-check OK**...

Results & Correlations: Z=50 and Z=20 isotopic chains

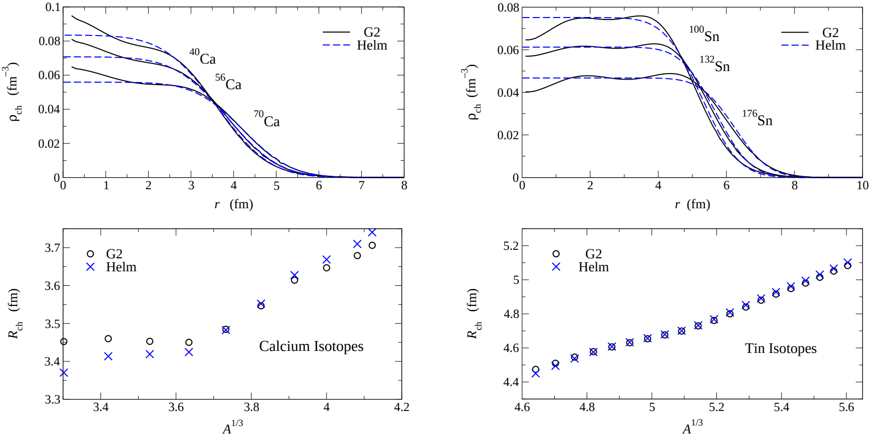
Charge Form Factor

F_{DWBA} increases and shifts towards smaller q as the neutron number increases

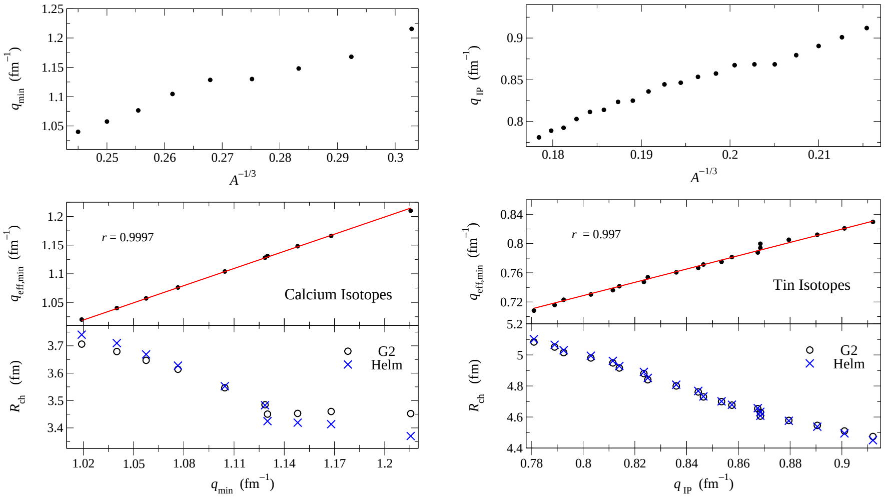


Methodology accurate for low-momentum transfer

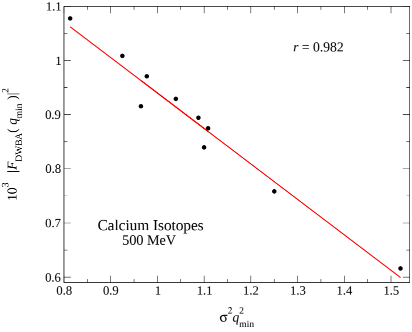
Helm and self-consistent charge densities and charge radii

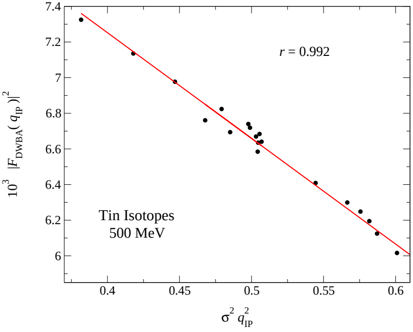


Correlations: evolution of first minimum or inflection point



Correlations: the smaller the bulk part of the nuclear charge distribution and the compact the surface, the smaller the form factor



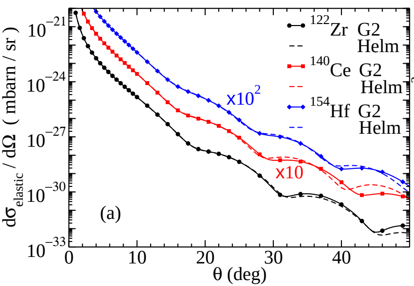


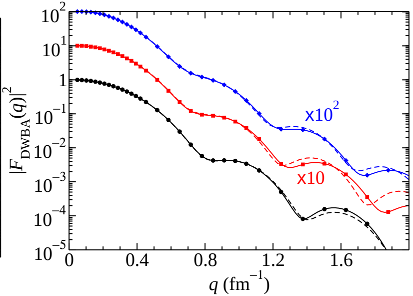
Therefore, if two or more isotopes have been measured ...

- linear correlations would provide, for an **unknown nucleus** of the chain, a **hint** on the value expected for the **square of the experimental electric charge form factor** at its first minimum
- if the value of the squared modulus of the **form factor is determined experimentally at its first minimum**, the **charge density in the Helm model can be sketched** from similar correlations
- use of **more elaborated versions of the Helm model** that take into account the central depression of the charge density should allow one to **extend the domain of validity of our method** up to larger values of the momentum transfer.

Results & Correlations: N=82, N=50 and N=14 isotonic chains

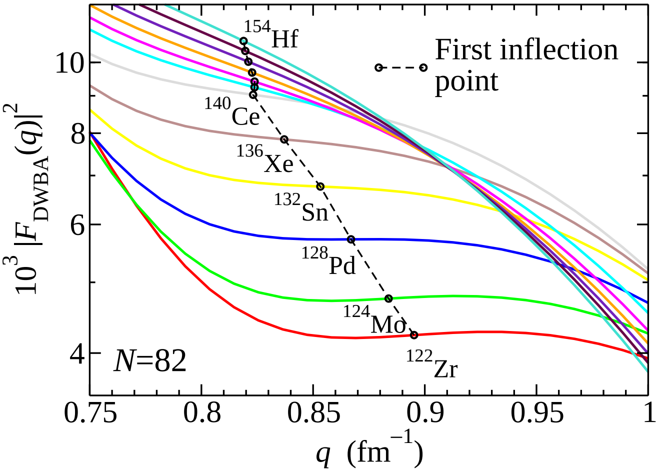
Differential cross sections and form factors



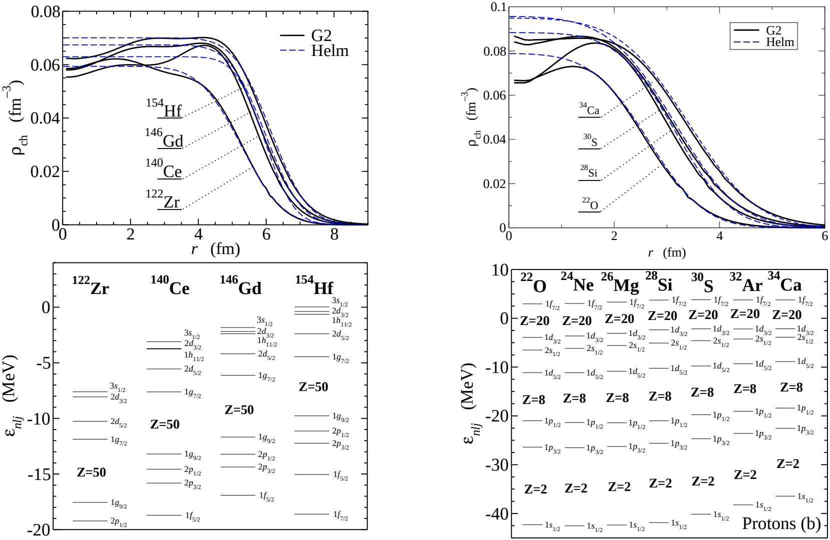


Methodology accurate for low-momentum transfer

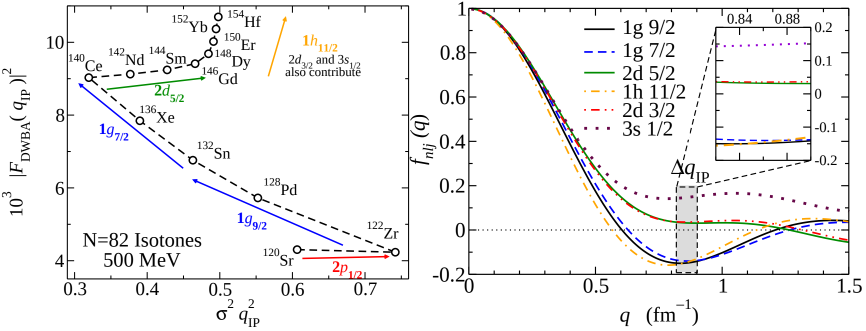
Charge form factors F_{DWBA} increases and shifts towards smaller q as the neutron number increases



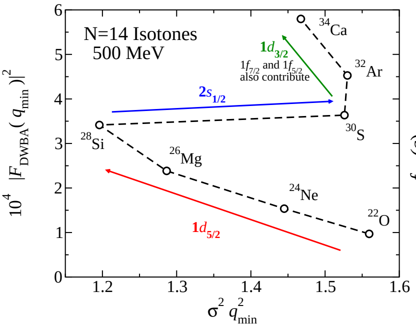
Charge densities and proton single particle levels

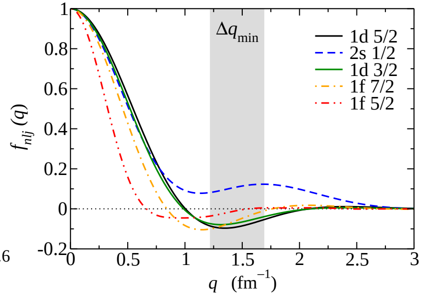


The increasing rate of the form factor basically depends on the proton level which is being filled!!



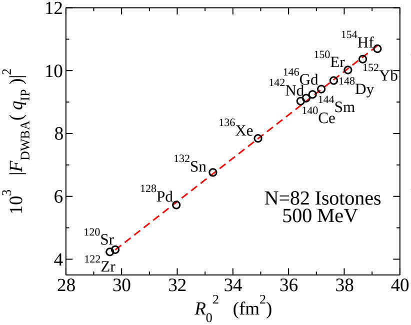
Also in lighter isotonic chains...

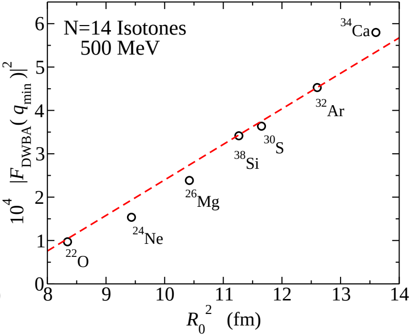




The larger the number of protons, the larger the formfactor

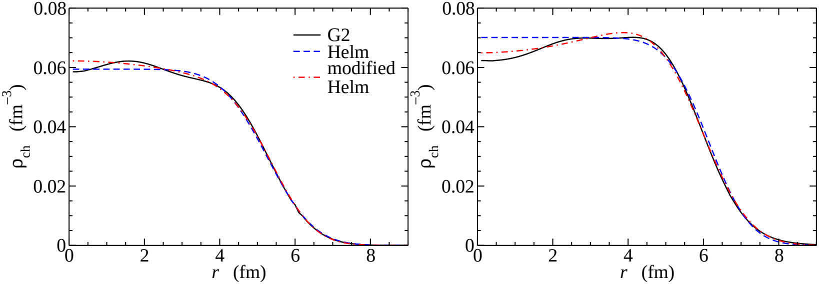
...this was clear, less clear was that it is almost linear along isotonic chains





What happens if we use a more realistic ansatz for the parametrized density?

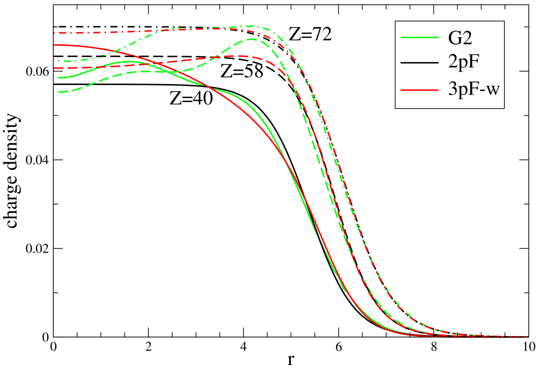
If we use a modified Helm density (3 parameters): able to reproduce the central "depression" or "bump" typically present in charge distribtions.



Just two examples of the $N = 82$ isotonic chain.

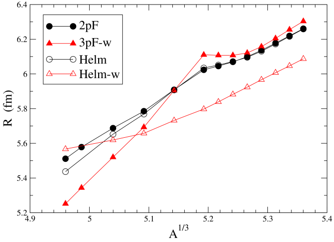
What happens if we use a more realistic ansatz for the parametrized density?

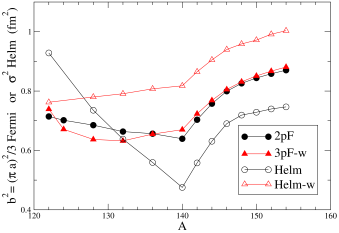
If we use a 2pF or 3pF density distributions.



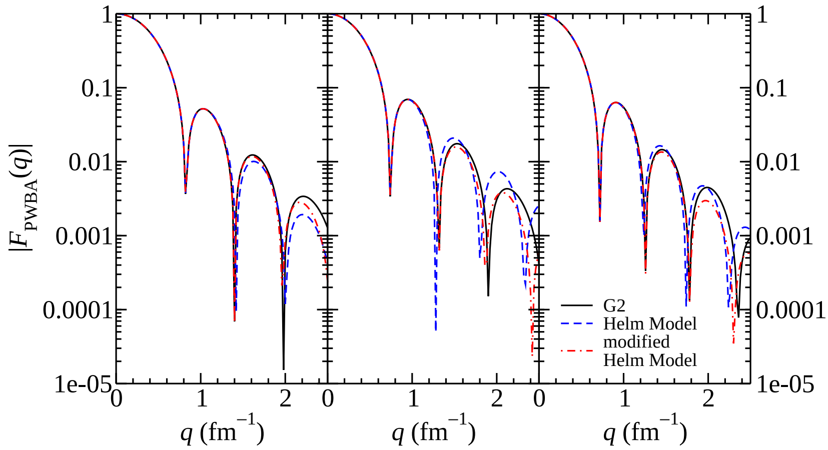
How much affects the “new” parameterized density our previous analysis?

One finds different quantitative results but **similar behaviours**
⇒ our analysis still valid when using other density distributions





How much the form factor looks like?



Conclusions: isotopic chains

- The described analysis is **potentially useful** for future electron-nucleus elastic **scattering experiments**,
  - the **linear correlations** shown would provide, for an unknown nucleus of a chain, a hint on the value expected for $|F_{\mathrm{exp}}(q_{\mathrm{min}})|^2$.
- The **exact analysis of the Coulomb phase shifts** applied to a exotic nuclei and compared with future measurments could, potentially, elucidate some aspects related with the **isospin asymmetry** of the nuclear force.
- The use of more **elaborated versions** of the Helm model should allow one to **extend** the domain of **validity** of our method up to **larger** values of q

Conclusions: isotonic chains

- Rate of change of the **electric charge form factor** is **extremely sensitive** on the **proton level which is being filled**
  - levels with large n and small l contribute with opposite sign with respect to levels without radial nodes and large angular momenta.
  - plotting $|F(q)|^2$ against $\sigma^2 q^2_{\mathrm{min}}$ magnifies such effects
- Therefore, **electron scattering** in isotonic chains can be a useful tool to **probe the proton single-particle shell structure of exotic nuclei**: filling order and occupancy of the different valence proton orbitals.

Conclusions: warning...

Extensive experimental investigations more difficult because of the limitations arising from **small production rates, short half-lives, and small cross sections** when one deals with unstable nuclei

Collaborators:

Xavier Viñas

Mario Centelles

Francesc Salvat

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Thank you for your attention!