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Renormalization of zero-range effective interactions in finite nuclei

Examples for a simplified Skyrme interaction

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- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different nuclear interactions in the medium** are found **depending** on the **approach**
  - EoS and (recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**[1]
- Based on effective interactions, **Self-consistent Mean-Field models** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...** and can be applied **along the whole nuclear chart** (expected to be **more accurate for heavy** than for light systems).

[1] Some heavier nuclei (Tin) have been also accurately predicted with the help of many-body (Oxigen) data [A. Ekström *et al.* Phys. Rev. C **91**, 051301(R) (2015)].

Mean field framework is particularly **suitable** for the study of ground state properties in **heavy** elements

- The **complexities of the system are effectively included into the interaction**, simplifying the wave functions using Slater determinants ($|\varphi\rangle$)
  $\Rightarrow$ **SCMF**
  $$E = \langle\psi|\hat{H}|\psi\rangle \equiv \langle\varphi|\hat{H}_{eff}|\varphi\rangle$$
- **Minimizing** the energy functional, **ground state properties are obtained**
- **Successful effective interactions**: **non-relativistic** **Skyrme** (zero-range) or **Gogny** (zero-range + finite range) and based on **relativistic** Lagrangians (both types)
- Effective interactions are usually **FITTED** to bulk properties of a set of nuclei and on empirical properties of nuclear matter

But note!

- Fitted **parameters** contain **correlations beyond the mean-field**
- We know the **description** of **single-particle states** and their spectroscopic factors is **not good** at the Mean-Field level.
- Mean-Field predicts a **low density of states** around the Fermi energy (m^*/m should ~ 1 at the nuclear surface and smaller in the interior)
- Only oscillation frequencies but **not widths of Giant Resonances** and other excited states are well reproduced.

... possible solution

- Many **corrections** to the **static Mean-Field** solution are **dynamical correlations** arising, for example, from the **coupling** with collective states (nuclear surface vibrations or **phonons**) [e.g. C. Mahaux et al., *Phys. Rep.***120**(1985)1; P.F. Bortignon et al. Phys. Rep. 30, 305 (1977); M. Baldo et al., JPG 42 085109 (2015)]
- Accounting for the **coupling of the vibrational and nucleonic** degrees of freedom **improves** the description of single particle properties and resonance widths.
- Microscopic calculations are **now feasible**, adding PVC on top of the Mean Field **[G. Colò et al. PRC 82 064307 (2010); E. Litvinova et al. PRC 73, 044328 (2006)]**

Beyond-mean-field and divergences

Theoretical scheme

On the one side ...

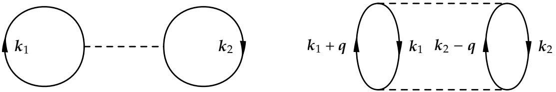
- If **beyond MF contributions** are computed, correlations are **explicitly considered** and a **refitting of the parameters is needed**
- **PVC** (specific type of BMF) calculations are commonly used with **non-refitted interactions** in the literature (thought some recipies exist)

On the other side ...

- If a **zero-range interaction** is used, **divergences** arise and a **renormalization** scheme is **needed**
- **Finite range** interactions will **not** suffer from **divergences** but matrix elements (**integrals**) are more **complicated** to calculate.

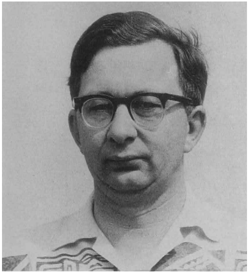
We consider ...

- the **lowest-order approximation BMF** [**PVC** in which the **phonon** is replaced by a **particle-hole (p-h) pair**]
- the **total energy** in both **nuclear matter** and **finite nuclei**



Diagrammatic representations of the **first-order** *and* **second-order** *total energy (for simplicity only direct terms are displayed)*

- **simplified version of the Skyrme** interaction in the vertex
- **cut-off** renormalization



Skyrme, *NPA* **9** (1959) 615

" . . . It is generally believed that the most important part of the two-body interaction can be represented by a contact potential . . . "
" ... this suggests an **expansion in powers of k and k' (initial and final relative momentum)**. If this expansion is stopped at the quadratic terms only a small number of undetermined coefficients occur, and an attempt can be made to determine these by comparison with experimental energies.... "

" ... on the other hand **this form is unrealistic for large momentum transfers**, so that it is not suitable for the discussion of **second-order effects**, unless some **momentum cut-off** is introduced ... "

$$V(\mathbf{r},\mathbf{r}',\mathbf{R},\mathbf{R}') = \frac{\sqrt{2}}{4} g\left(\frac{\mathbf{R}}{\sqrt{2}}\right) \delta(\mathbf{r})\delta(\mathbf{r}')\delta(\mathbf{R}-\mathbf{R}') \quad \text{where} \quad g = t_0 + \frac{t_3}{6}[\rho(\mathbf{R})]^\alpha$$

(spin-dependent, velocity-dependent, and spin-orbit terms are dropped) $\mathbf{r} \equiv (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$ and $\mathbf{R} \equiv (\mathbf{r}_1 + \mathbf{r}_2)/\sqrt{2}$

$$v^{\lambda\lambda'}(\boldsymbol{r}',\boldsymbol{r}) = \frac{1}{\Omega}\int \mathrm{d}_3k\mathrm{d}_3k'\, \mathrm{e}^{i\boldsymbol{k}'\cdot\boldsymbol{r}'} v\left(\boldsymbol{k}',\boldsymbol{k}\right)\theta(\lambda-k)\theta(\lambda'-k')\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$$
$$= \frac{1}{4\pi^4}\frac{\lambda^2\lambda'^2}{rr'} j_1(r\lambda)j_1(r'\lambda') \xrightarrow{\lambda,\lambda'\to\infty} \delta(\mathbf{r})\delta(\mathbf{r}')$$

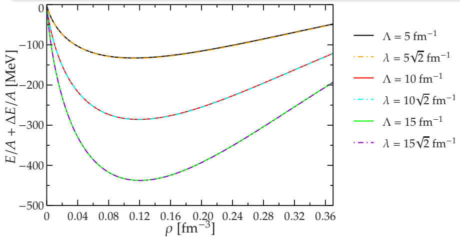
- The system is **translationally invariant**
  - Wave-functions are **plane waves**
  - **Momentum is conserved**
  - **Transferred momentum ($\mathbf{q} = (\mathbf{k}' - \mathbf{k})/\sqrt{2}$)** is a good quantum number (rel. momentum $\mathbf{k} \equiv (\mathbf{k}_1 - \mathbf{k}_2)/\sqrt{2}$)
- **Potential** energy of the system **[interaction strength $g = t_0 + \frac{t_3}{6}\rho^\alpha$]**
  - **First order** (HF):
    $$E = \frac{1}{2}\sum \langle ij|\bar{V}|ij\rangle \rightarrow \frac{E}{A} = \frac{d\Omega^2}{A(2\pi)^6}\int_{k_1,k_2<k_F} d^3k_1 d^3k_2 v(k_1,k_2) = \frac{3}{8}g\rho$$
    First order with the cutoff on relative momentum $k < \lambda$:
    $$\frac{E}{A} = \frac{3}{8}g\rho\left(8\beta^3 - 9\beta^4 + 2\beta^6\right) \text{ where } \beta \equiv \min\{1, \lambda/(\sqrt{2}k_F)\}$$
  - **Second order**: $\Delta E = \frac{1}{2}\sum \frac{\langle mn|V|ij\rangle\langle ij|\bar{V}|mn\rangle}{\varepsilon_i + \varepsilon_j - \varepsilon_m - \varepsilon_n} \rightarrow$
    $$\frac{\Delta E}{A} = \frac{d\Omega^3}{A(2\pi)^9}\int_{k_1,k_2,q} \frac{v^2(k_1,k_2,q)d^3k_1 d^3k_2 d^3q}{\varepsilon_{k_1} + \varepsilon_{k_2} - \varepsilon_{k_1+q} - \varepsilon_{k_2-q}} \left(\sim v^2\int \frac{d^3q}{q^2}\right)$$

    diverges linearly with q

    The **integral** can be calculated with the **cutoff in the relative momentum** $k, k' < \lambda = \lambda'$ and with the cutoff in the **transfered momentum** $q < \Lambda$: both results coincide for large values of the cutoff ($\lambda > 2\sqrt{2}k_F$ where $\lambda = \sqrt{2}\Lambda$)

In NM there exist two well defined equivalent momenta that can be cut !
1) Initial and final **relative momentum** (**k**) between interacting particles OR
2) the **momentum transfer** (**q**) from initial to final states. Their relation:

$$\boxed{\vec{q} = (\vec{k}' - \vec{k})/\sqrt{2}}$$

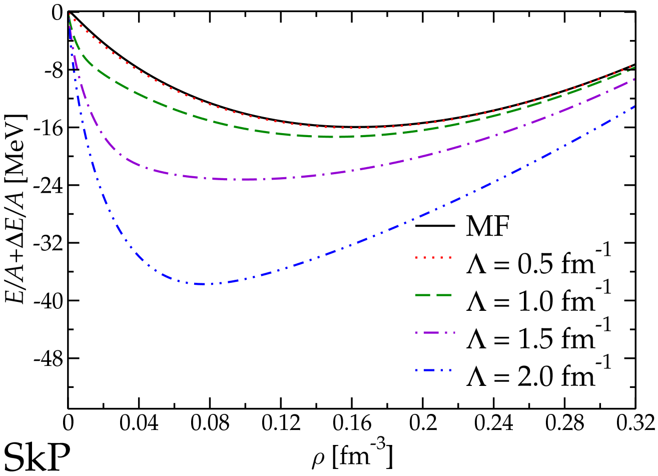


* The **relation** between **cut-offs** is still to be fully **clarified**
* The **comparison here is numerical** since the analytic expressions are hard to derive
* The **non-divergent** contributions (analytic) are **identical**

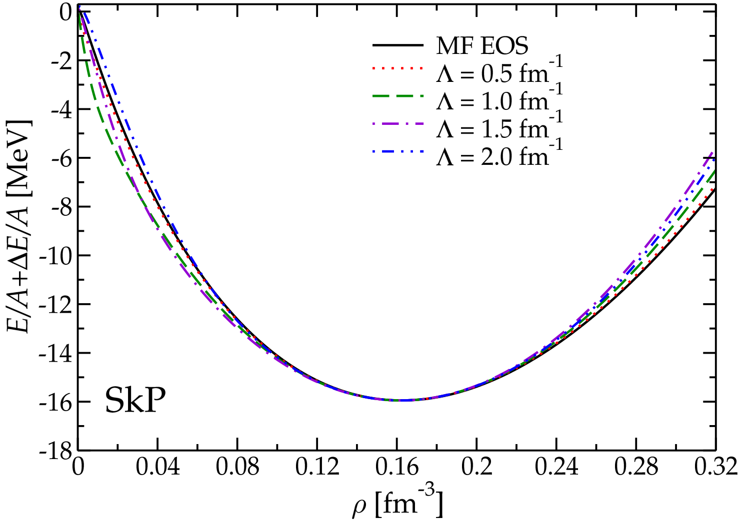
Which is the physical range for the momentum cutoff? from **0** to **?**

- Typical distance between nucleons in the nucleus: $2r_0 \sim 2 \times 1.2$ fm $\rightarrow pc \sim 2\pi\hbar c/2r_0 \sim \boxed{1.3 \text{ fm}^{-1}}$
- Mass of the "Goldstone" boson carrying the interaction: the pion 140 MeV$/\hbar c \sim \boxed{0.7 \text{ fm}^{-1}}$
- Energy needed to excite the nucleon: $m_\Delta - m_{\text{nucl}} \sim \boxed{1.5 \text{ fm}^{-1}}$

In K. Moghrabi *et al.* PRL **105** 262501 (2010): Cutoff on transferred momentum ($q < \Lambda$); $t_0 - t_3$ model; SkP as reference EoS (since $m^*/m = 1$)



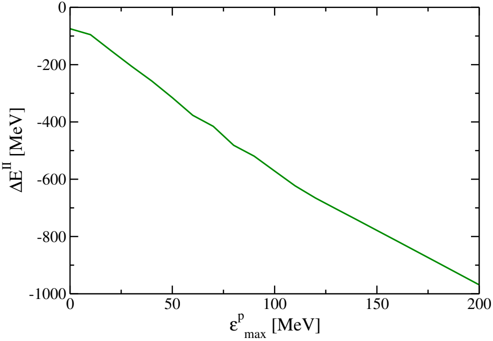
Nuclear matter
Total energy renormalized



Towards finite systems

- Computation of the total energy at second order of a finite nucleus: as a **test we choose ^{16}O**
- Simplified model with only $t_0 - t_3$ **part of the Skyrme** interaction

$$\boxed{\Delta E = \frac{1}{2} \sum \frac{\langle mn|V|ij\rangle\langle ij|\bar{V}|mn\rangle}{\varepsilon_i + \varepsilon_j - \varepsilon_m - \varepsilon_n}}$$



The **divergence increases lineraly** with increasing **model space** (maximum particle state energy). **Cutoff not implemented yet.**

- **Use the interactions SkP$_\Lambda$** fitted in symmetric nuclear matter for each cutoff to check if the divergence is cured.
- **Absence of translational invariance**
  - **No momentum conservation**
- The **Skyrme interaction is a low-momentum expansion** of the effective nuclear potential going up to second order in **relative momenta** $\Rightarrow$ **cutoff** on **relative momenta** [B. G. Carlsson *et al.* PRC **87** 054303 (2013)]
  - We will use relative and center-of-mass coordinates
- The **cutoff** is included **also at mean-field** level

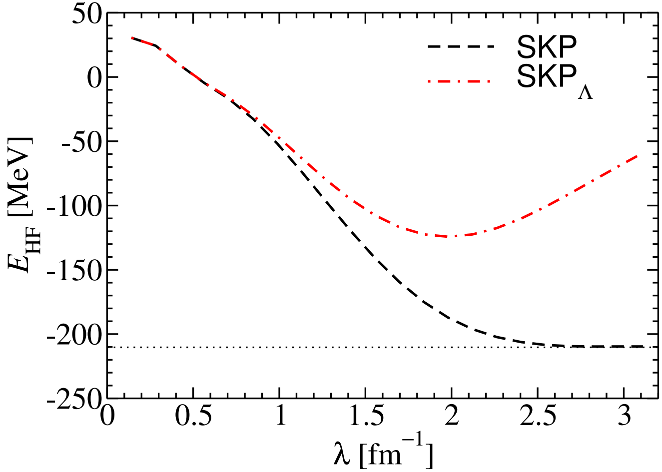
Interaction in terms of the cutoffs $\lambda = \lambda'$ on initial and final relative momenta:

$$v^{\lambda\lambda'}(\boldsymbol{r}', \boldsymbol{r}) = \frac{1}{\Omega} \int \mathrm{d}_3 k \mathrm{d}_3 k' \, \mathrm{e}^{i\boldsymbol{k}'\cdot\boldsymbol{r}'} v\left(\boldsymbol{k}', \boldsymbol{k}\right) \theta(\lambda - k)\theta(\lambda' - k')\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$$
$$= \frac{1}{4\pi^4} \frac{\lambda^2 \lambda'^2}{r r'} j_1(r\lambda) j_1(r'\lambda')$$

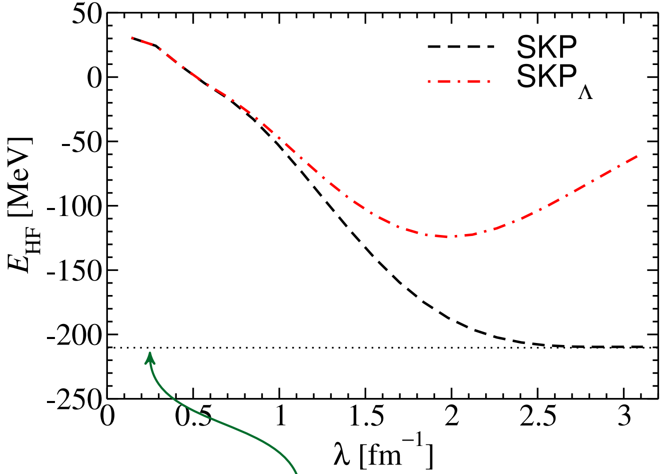
"The interaction acquires a finite range"

To compute the **matrix elements** of the interaction ...

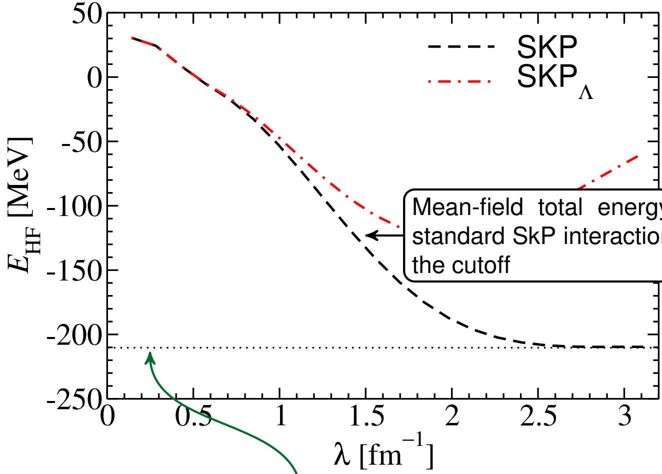
- ... we expand the **sp wave functions** on a **HO basis** and transform **initial and final two-particle states to the CM and relative motion coordinates** (Brody-Moshinsky)
- **Solve HF** equations **on a HO basis**, using the matrix elements of $v^{\lambda\lambda'}$
- Computation of the **second order** total **energy** with the same **interaction** and **cutoff** used at mean-field level.
- The **interaction** used is the **same at each step** and we have no free parameters, but the second order correction is added **perturbatively**.
- **No self-consistency** since the equations are not solved iteratively.



SKP
SKP$_\Lambda$
E_{HF} [MeV]
λ [fm^{-1}]
50
0
-50
-100
-150
-200
-250
0
0.5
1
1.5
2
2.5
3



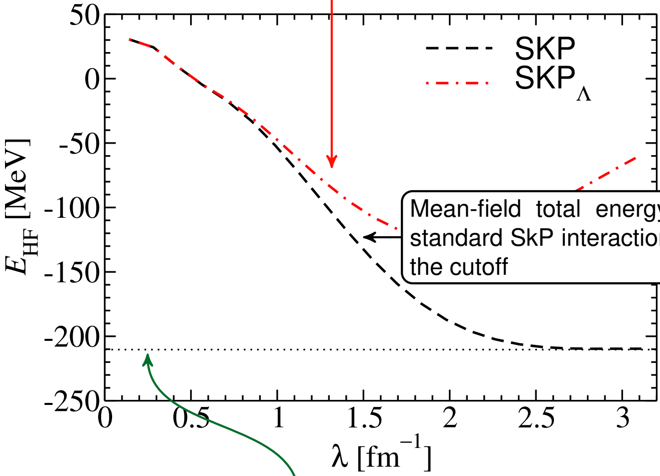
Mean-field total energy with the standard SkP interaction, without any cutoff



Mean-field total energy with the standard SkP interaction *including* the cutoff

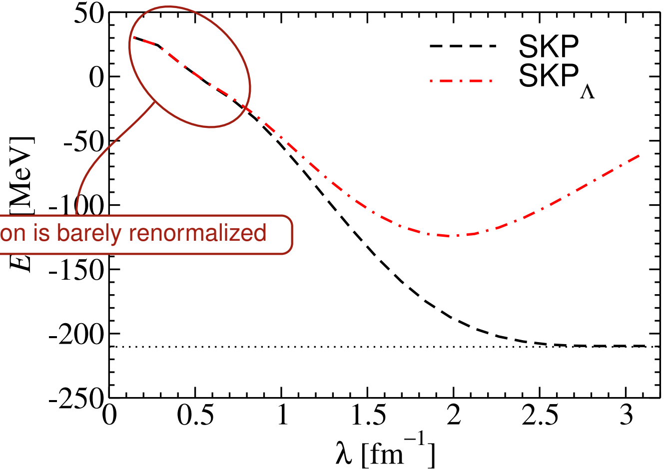
Mean-field total energy with the standard SkP interaction, without any cutoff

Mean-field total energy with the *renormalized* SkP interaction *including* the cutoff

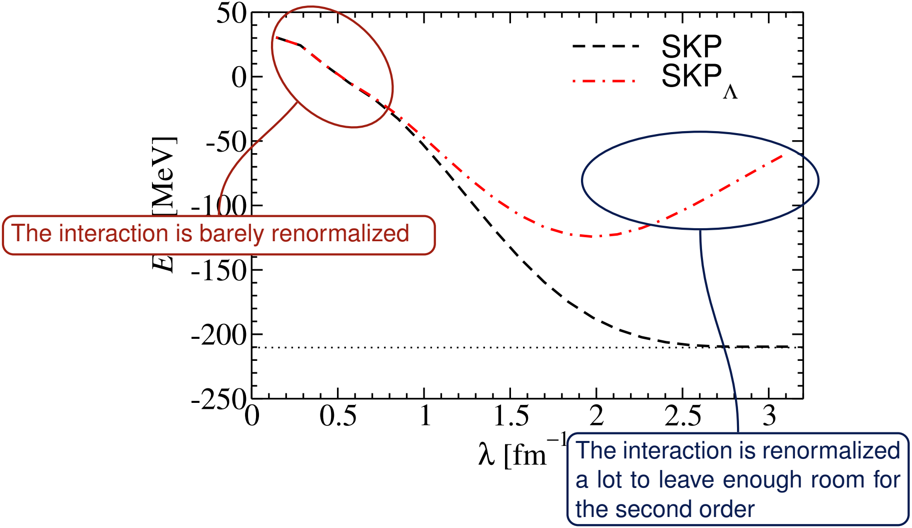


Mean-field total energy with the standard SkP interaction *including* the cutoff

Mean-field total energy with the standard SkP interaction, without any cutoff

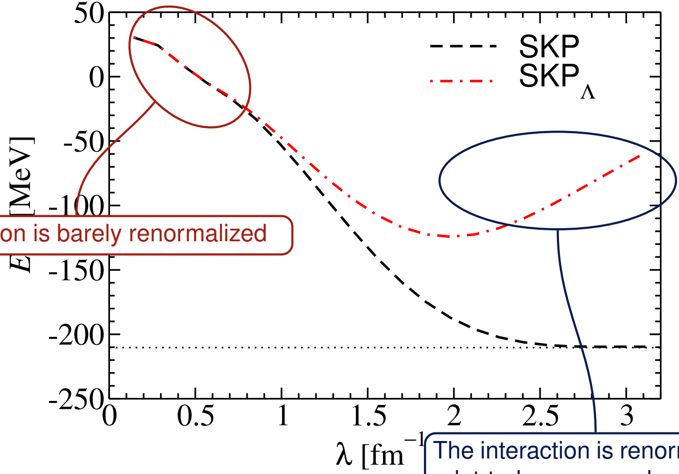


The interaction is barely renormalized



50
0
-50
-100
-150
-200
-250
0
0.5
1
1.5
2
2.5
3
λ [fm^{-1}]
E [MeV]
SKP
SKP$_\Lambda$
The interaction is barely renormalized
The interaction is renormalized a lot to leave enough room for the second order

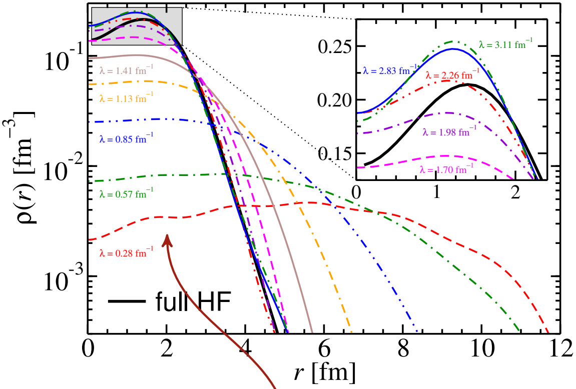
The mean-field results
Total energy – ^{16}O



The interaction is barely renormalized

The interaction is renormalized a lot to leave enough room for

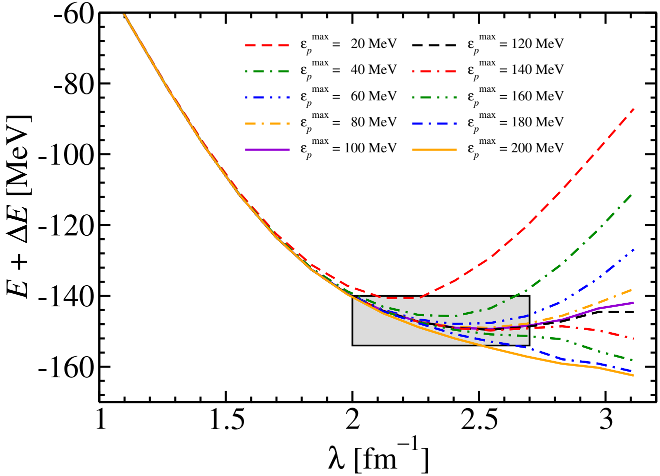
We expect that in these regions the problem cannot be treated perturbatively



If the cutoff is small the system resembles a gas of nucleons: the mean square radius when $\Lambda = 0.28\,\mathrm{fm}^{-1}$ is ≈ 10 fm

The total energy at second order

Refitted interactions ^{16}O



Total energy constant within a ~ 10 %

- **Mean Field**
  - provides an approximate realization of an **energy density functional**
  - **suitable** for the study of masses, radii, deformation, ... along the **whole nuclear chart**
  - **sp properties** are not within its domain: an extension is required
- **Beyond Mean Field**
  - **different approaches exist: not a unique and systematic way to improve the method/s and to develop renormalization procedures**
  - **PVC** provides a better description of sp and collective phenomena
  - we tried to develop a **simple model based** on the PVC idea but substituting the phonons by $p-h$ pairs. We **address** the problem of **renormalizability of a zero-range interaction** in **NM and FN**.
- **In more detail, ...**
  - the **second order corrections** to the total energy have been included **perturbatively**
  - total energy of ^{16}O with a **simplified Skyrme interaction**
  - use the **interaction fitted in nuclear matter for each cutoff**
  - cutoff **renormalization feasible** on the relative momenta
  - **window** in λ and $\varepsilon_p^{\text{max}}$ where the **total energy is constant** within 10-15%

We aim at developing a fully self-consitent method BMF, that can be systematically improved and renormalized, and that allow us to study nuclear phenomena along the whole nuclear chart. Some of the main steps to be done are to ...

- treat the **continuum exactly**, in part also to avoid model dep. on $\varepsilon_p^{\mathrm{max}}$
- include **pairing** in our formalism
- solve **self-consistently the Dyson equation** for the sp propagator up the required order
- **refit** the parameters at the adopted level of app. by using FN data
- include the **full Skyrme** interaction and/or propose a **new interaction**
- explore different **renormalization** schemes such as the dimensional regularization

Collaborators

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