Nuclear equation of state from nuclear collective excited state properties

Xavier Roca Maza
Università degli studi di Milano & INFN

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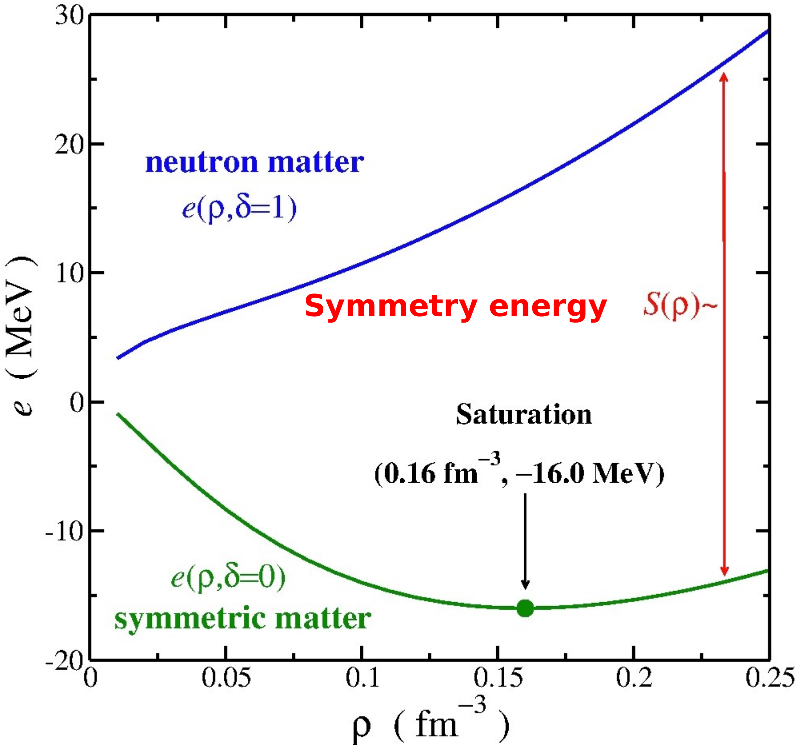
The nuclear Equation of State

Unpolarized **nuclear matter** at zero temperature (10^{10}K→1MeV) is defined as the **energy** per **nucleon** (*e*) as a function of the **neutron** (ρ_n) and **proton** (ρ_p) **densities** as (*isospin conserving* $V_{nn} = V_{pp} = V_{np}$):

$$\boxed{e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2} + \mathcal{O}[\delta^4] \quad \text{where } \rho = \rho_n + \rho_p \text{ and } \delta = \frac{\rho_n - \rho_p}{\rho}$$



It is customary to **expand *e(ρ,δ)*** around nuclear **saturation density** $\rho_0 \sim 0.16\ \text{fm}^{-3}$

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0 x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

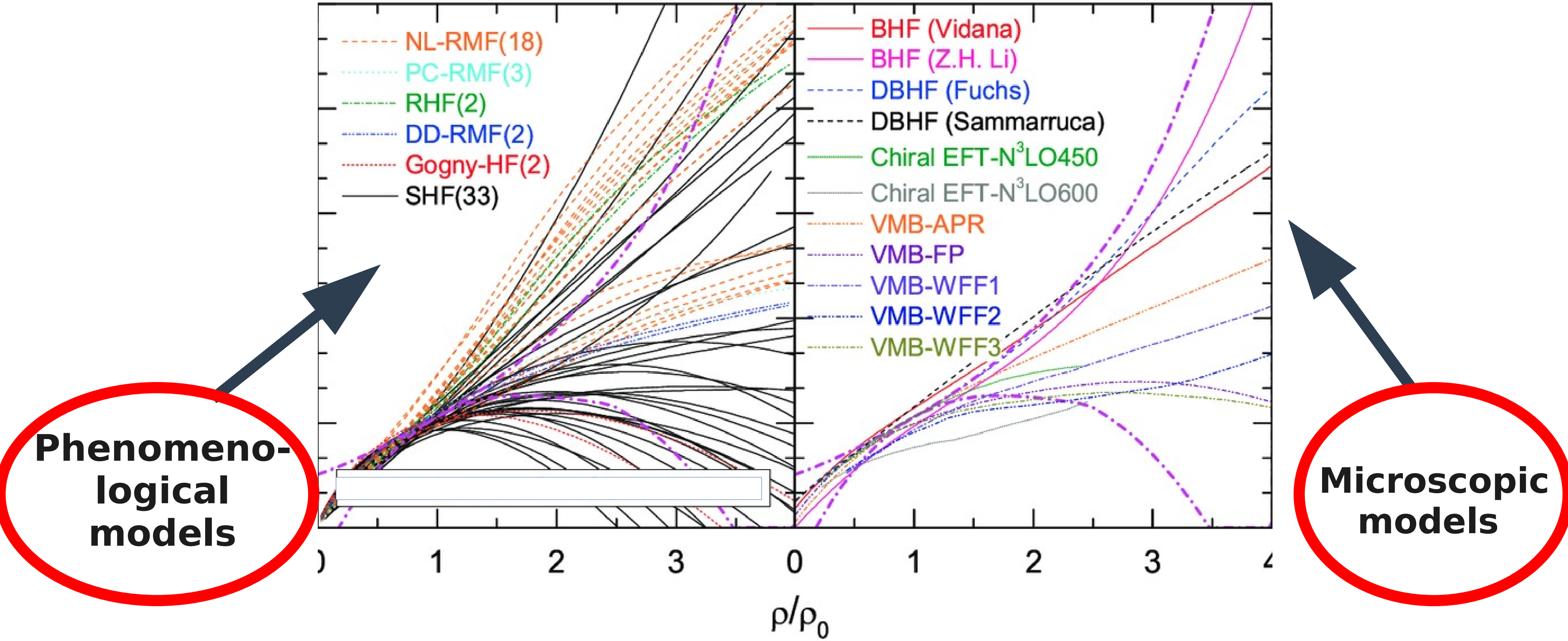
K_0 → how **compressible** is symmetric matter at ρ_0

J → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L → **neutron pressure** in neutron matter at ρ_0

Symmetry energy $S(\rho) \sim e(\rho,\delta=1)-e(\rho,\delta=0)$

Symmetry energy not well constrained

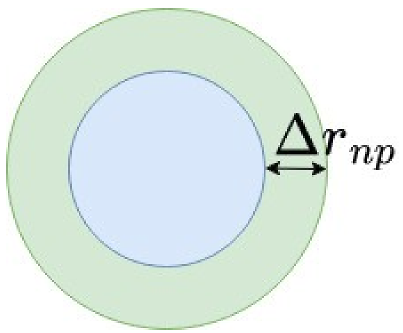


Towards understanding astrophysical effects of nuclear symmetry energy
Bao-An Li, Plamen G. Krastev, De-Hua Wen & Nai-Bo Zhang EPJ A 55, 117 (2019)

Neutron skin (g.s.) as a proxy to L

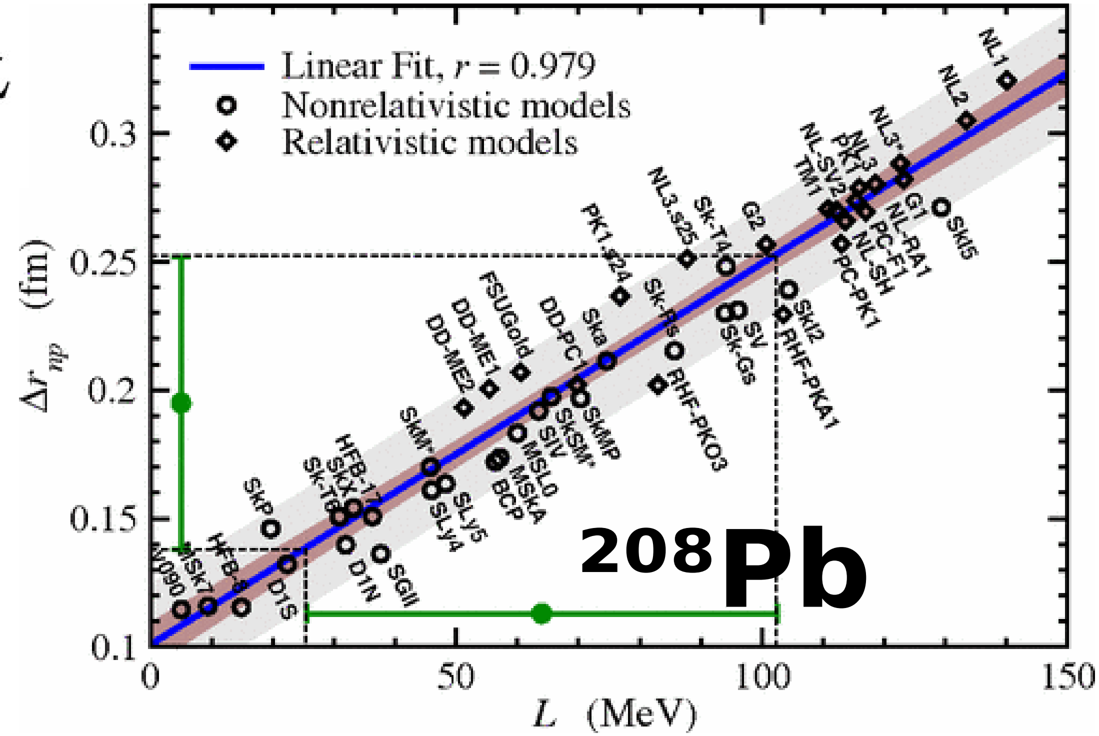
The **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in a heavy neutron rich nucleus is related to the **neutron pressure** ($\delta=1$) around ρ_0 (L).

$$P(\rho_0,\delta)=\rho_0^2\frac{\partial e(\rho,\delta)}{\partial\rho}\bigg|_{\rho=\rho_0}=\frac{1}{3}\rho_0\delta^2 L$$



→ From the Droplet Model:

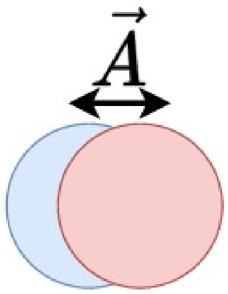
$$\Delta r_{np}\approx\frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

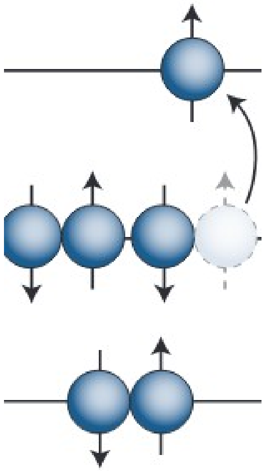


Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

Giant Resonances

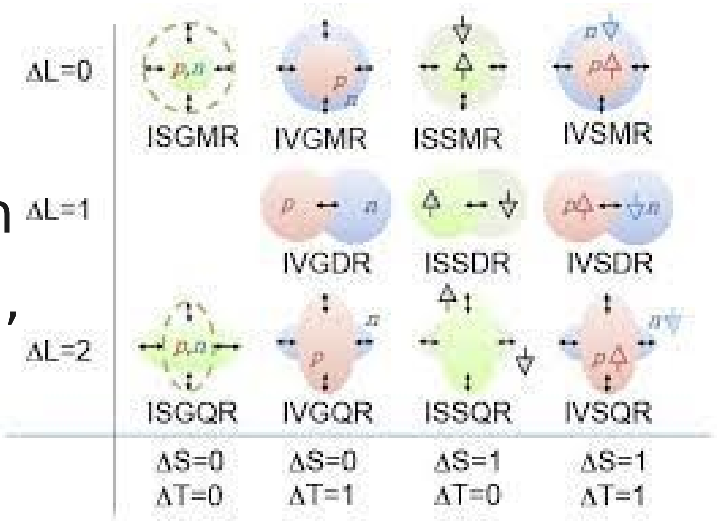
→ **Collective motion** giving rise to **Giant Resonances** could be **qualitatively** understood as *macroscopic* **oscillation/vibration** of the **nucleon densities** $\delta\rho \sim \mathbf{A}\cdot\nabla\rho$





→ **Microscopically,** overall Giant Resonance properties have been seen to be well described as a **coherent superposition** of **1 particle – 1 hole** excitations.

→ **Collective excitations** can be **classified** in terms of the quantum numbers $\mathbf{J} = \mathbf{L} + \mathbf{S}$ and **parity**, $\pi=(-1)^L$, and modeled by using transition operators of the type $\sum \tau_a\, r^L\, (\mathbf{Y}_{LM} \times \boldsymbol{\sigma})_{JM}$. with τ_a=1, τ_z for non-charge exchange IS and IV, respectively, or $\tau_a=\tau_\pm$ for charge exchange.



Sum rules

→ **Sum Rules** or moments of the strength function S(E) sometimes **model independent** ([F,V]=0) and/or predict **very simple expressions** that can be exploited to better **understand the nuclear phenomenology.**

k-moment of S(E):

$$m_k \equiv \int dE E^k S(E)$$

$$m_0 = \sum_\nu |\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger F|0\rangle$$

$$m_1 = \sum_\nu (E_\nu - E_0)|\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger[\mathcal{H}, F]|0\rangle$$

$$m_2 = \sum_\nu (E_\nu - E_0)^2|\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger[\mathcal{H}, [\mathcal{H}, F]]|0\rangle$$

$$m_3 = \sum_\nu (E_\nu - E_0)^3|\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger[\mathcal{H}, [\mathcal{H}, [\mathcal{H}, F]]]|0\rangle$$

Thouless Th→
m_1(RPA) =1/2
<HF|D.C.|HF>

Expectation value of commutators in the G.S.!!

Example for non-spin flip resonances:

$$m_1^{IS} = \frac{A}{4\pi}\frac{\hbar^2}{2m}\frac{2J+1}{A}\int d\vec{r}\left[\left(\frac{df}{dr}\right)^2 + J(J+1)\left(\frac{f}{r}\right)^2\right]\rho(r)$$

$$m_1^{IV} = m_1^{IS}(1+\mathcal{K})$$

$\boxed{f = r^L}$

κ ↔ [F,V] non zero

Sum rules: dielectric theorem

Ground state perturbed by an external field λF (λ → 0) so that perturbation theory holds → The **expectation value of the Hamiltonian** <H> :

$$\delta\langle\mathcal{H}\rangle = \lambda^2 \sum_{\nu\neq 0} \frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^3) = \lambda^2 m_{-1} + \mathcal{O}(\lambda^3)$$

$$\delta\langle F\rangle = -2\lambda \sum_{\nu\neq 0} \frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^2) = -2\lambda m_{-1} + \mathcal{O}(\lambda^2)$$

$$\boxed{m_{-1} = \frac{1}{2}\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\lambda^2}\bigg|_{\lambda=0} = -\frac{1}{2}\frac{\partial\langle F\rangle}{\partial\lambda}\bigg|_{\lambda=0}}$$

Use of sum rules: examples

Agree with thermodynamic definition of $1/\chi$

→ Calculate the finite nucleus incompressibility

$$(E_x^{\mathrm{ISGMR}})^2 = \frac{m_1}{m_{-1}} = 4\frac{\hbar^2}{m}\langle r^2\rangle \frac{\partial^2 E}{\partial\langle r^2\rangle^2} = 4A\frac{\hbar^2}{m\langle r^2\rangle}\langle r^2\rangle^2\frac{\partial^2(E/A)}{\partial\langle r^2\rangle^2} \equiv K_A\frac{\hbar^2}{m\langle r^2\rangle}$$

→ **Connection with EoS**: Blaizot model frequently used to connect K_A with K_{volume} (**<u>qualitative!;</u> systematic theoretical errors may be large**)

$$K_A = K_{\mathrm{vol}} + K_{\mathrm{surf}}A^{-1/3} + K_\tau I^2 + K_{\mathrm{Coul}}Z^2A^{-4/3} + \ldots,$$

→ **Alternatively:** the minimum condition of **monopole** (isotropic) **dilation** on the g.s. $\Psi(r) \rightarrow \alpha^{3/2}\,\Psi(\alpha r)$ leads to $\mathbf{E(\alpha) = \langle\Psi(\alpha)|H|\Psi(\alpha)\rangle}$ which can be used to calculate m_3 (scaling approach with $E^2 = m_3/m_1$):

$$(E_x^{\mathrm{ISGMR}})^2 = \frac{\hbar^2}{mA\langle r^2\rangle}\left[4T + 9E_{\mathrm{contact}} + 25E_{\mathrm{SO}} + \ldots\right]$$

→ However "safetiest" → direct comparison of measured observable with theory → same theory to predict K_{volume}

Use of sum rules: example for guidance

→ One can calculate the **polarizability**, proportional to m_{-1}, from the **dielectric theorem** and **Droplet Model:**

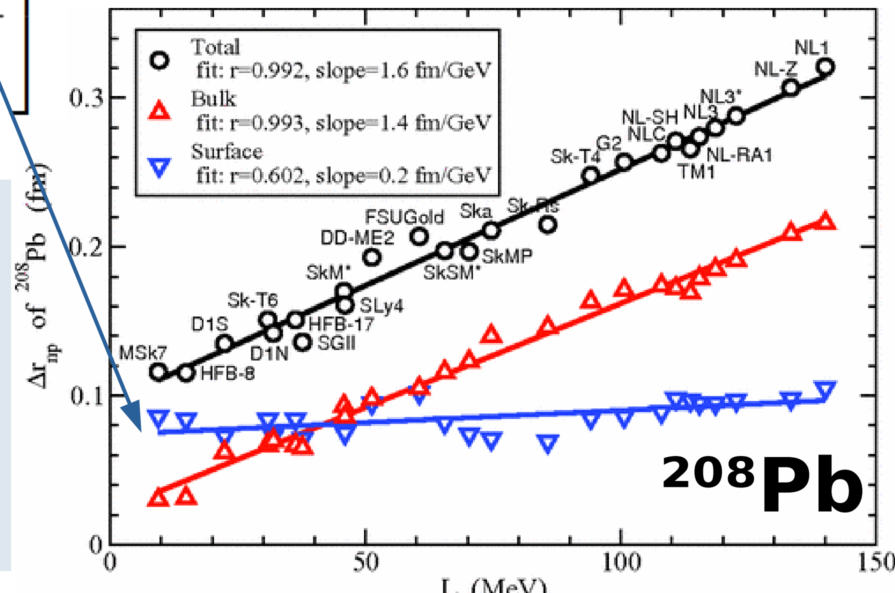
$$\alpha_D = \frac{8\pi e^2}{9} m_{-1}(E1)$$

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

Using definition of α and D.M. relation between Δr_{np} and J/Q

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{surface}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

In stable nuclei, surface well constrained by fit to B and R_{ch}



Polarizability increases with the mass (for the dipole $A^{5/3}$, for the quadrupole $A^{7/3}$ and so on …) and **slope of the symmetry energy (L)** and **decrease** with the **bulk symmetry energy (J)** → **sets a relation between the EoS parameters J and L**

Use of sum rules: examples

→ Spin Dipole Resonance

$\sum_{i=1}^{A} \sum_{M} \tau_{\pm}(i) r_i^L \left[Y_L(\hat{r}_i) \otimes \boldsymbol{\sigma}(i) \right]_{JM}$

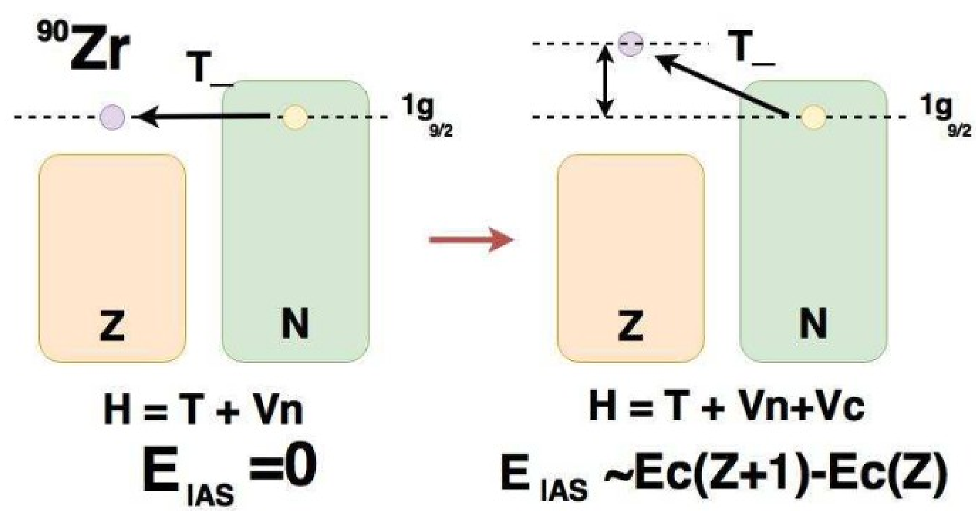
$$m_0(\tau_-) - m_0(\tau_+) = \frac{9}{4\pi}\left(N\langle r_n^2\rangle - Z\langle r_p^2\rangle\right)$$

$$\approx (N-Z)\langle r_p^2\rangle \left(1 + \frac{2N}{N-Z}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right)$$

Clear and direct relation with skin

→ Isobaric Analog State (m_1/m_0)

$\sum_{i=1}^{A} \tau_{\pm}(i)$



$$E_{IAS} = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle}$$

[H,T–] non zero only for ISB

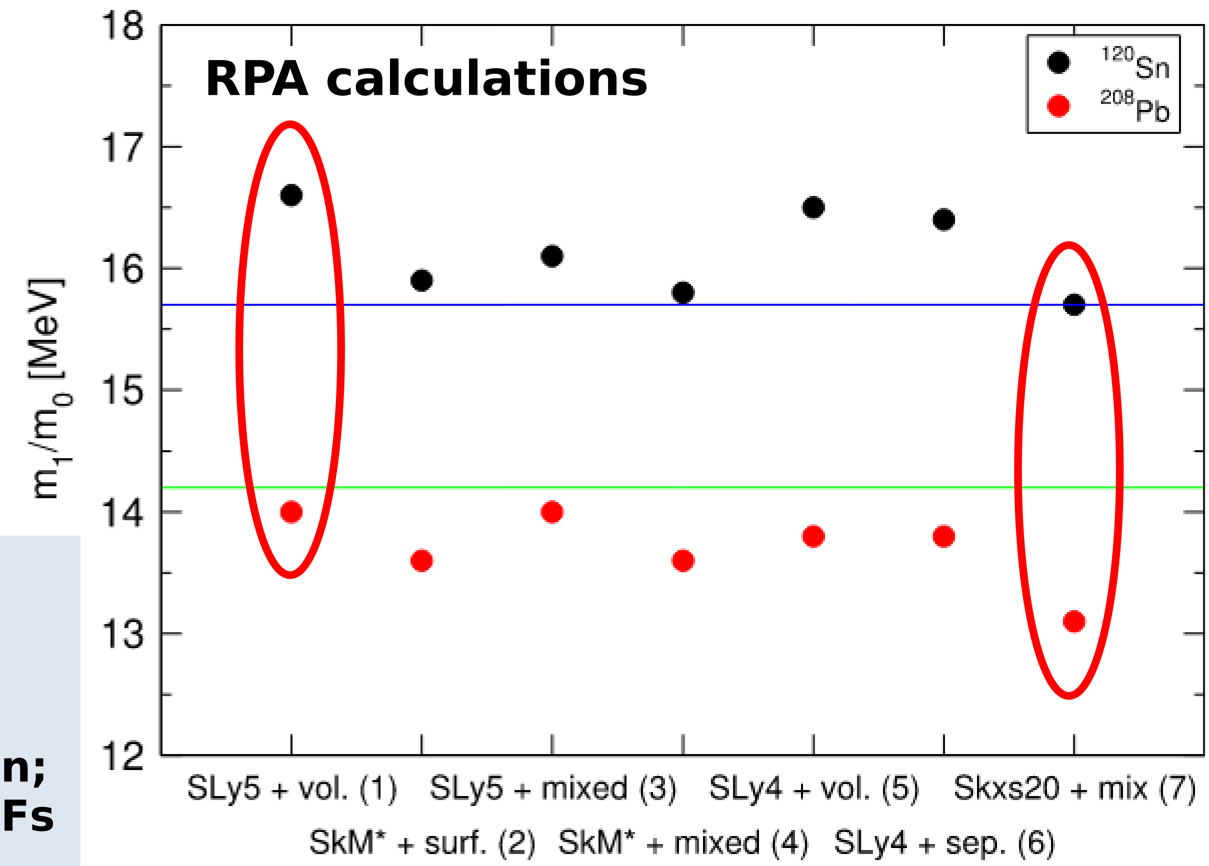
Relation with neutron skin

→ Independent particle picture & just Coulomb direct & sharp sphere app.:

$$E_{IAS} \approx E_{IAS}^{C,direct} \approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta r_{np}}{R_p}\right)$$

Non-charge exchange Giant Resonances: some examples

Giant Monopole Resonance:
do we understand it?



See Umesh Garg Talk!

Exp.

K_0<**240** MeV

(<u>Open shell</u>)

Exp.

K_0=**240±20** MeV

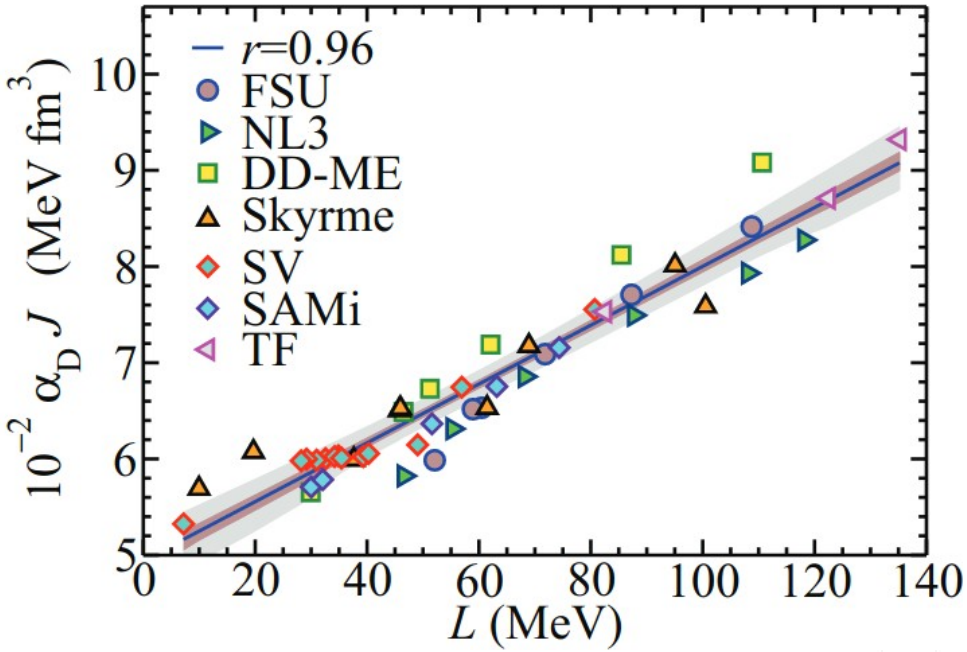
(<u>Closed shell</u>)

Pairing effects studied here

Further studies are needed (uncertainty quantification; Extended EDFs & ab initio)

U. Garg, G. Colò / Progress in Particle and Nuclear Physics 101 (2018) 55–95

Dipole polarizability



$$J = 25.0(2) + 0.19(2)L \quad \text{for } ^{68}\text{Ni},$$
$$J = 25.4(1.1) + 0.17(1)L \quad \text{for } ^{120}\text{Sn},$$
$$J = 24.5(8) + 0.17(1)L \quad \text{for } ^{208}\text{Pb}\,.$$

$$S(\langle\rho\rangle) \approx J - L\frac{\rho_0 - \langle\rho\rangle}{3\rho_0} \rightarrow J \approx S(\langle\rho\rangle) + \frac{\rho_0 - \langle\rho\rangle}{3\rho_0}L$$

(Warning: higher order terms might be relevant)

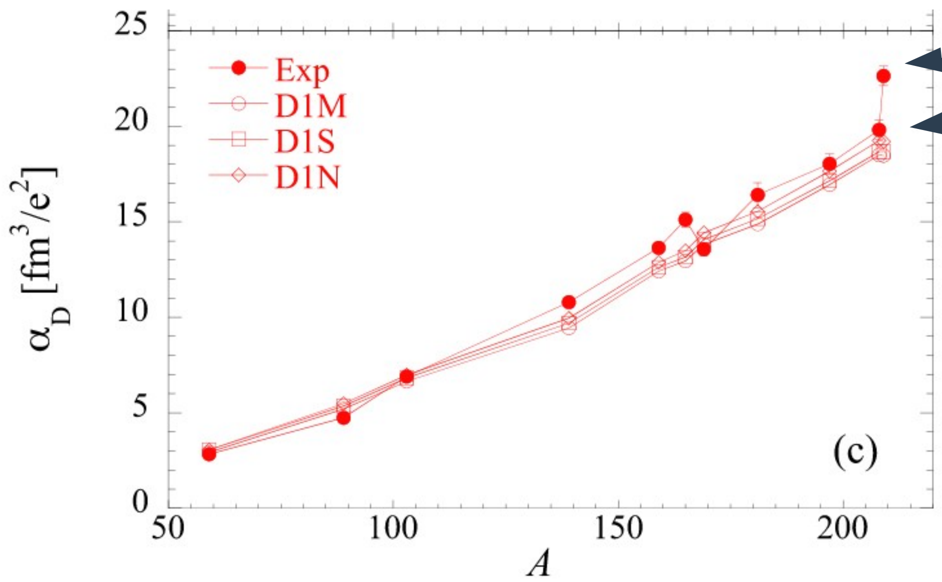
$$S(\langle\rho\rangle \approx 0.08\ \text{fm}^{-3}) \approx 25\ \text{MeV}$$

X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013

X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

Dipole polarizability:
do we understand it?

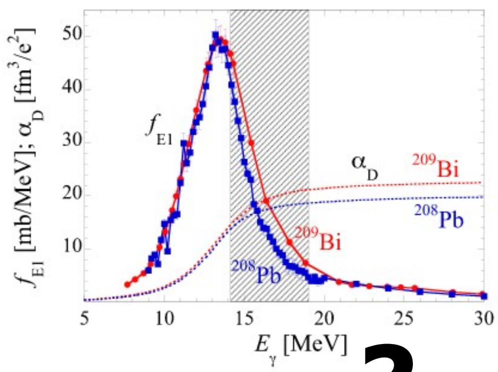
S. Goriely, S. Péru, G. Colò, X. Roca-Maza, I. Gheorghe, D. Filipescu, and H. Utsunomiya
Phys. Rev. C **102**, 064309 – Published 9 December 2020



^{209}**Bi**

^{208}**Pb**

?

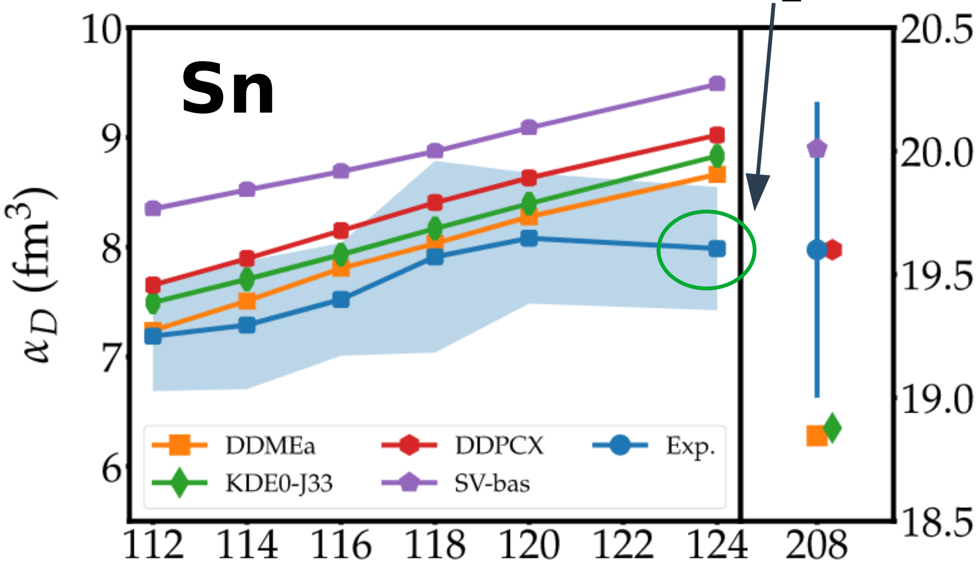


?

$$J = 32.5(1.0) + 0.17(0.02)L \quad \text{for } ^{89}\text{Y},$$

$$J = 26.7(6) + 0.14(0.02)L \quad \text{for } ^{139}\text{La},$$

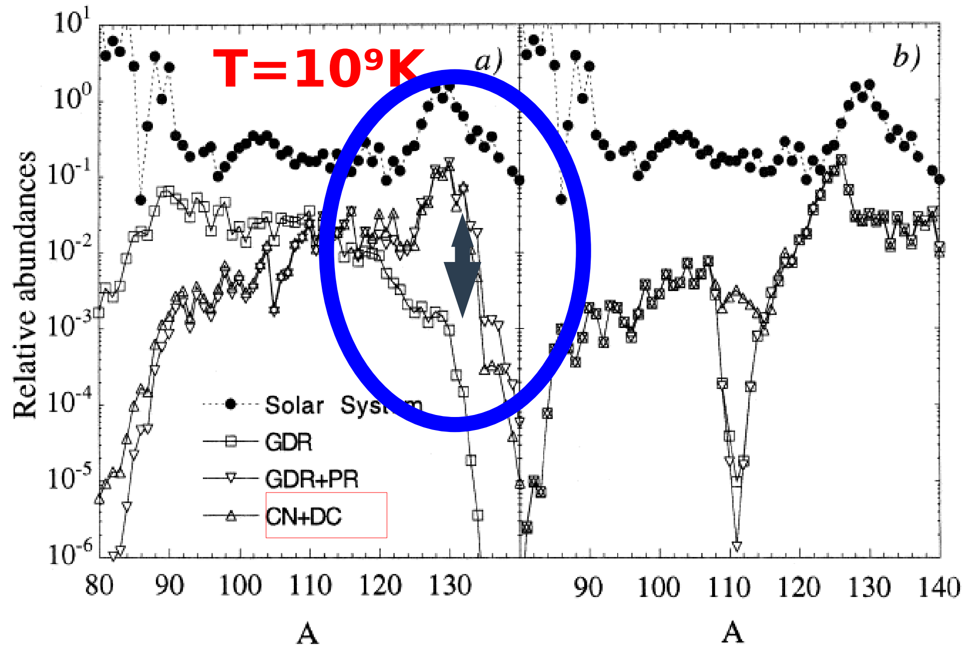
$$J = 24.0(6) + 0.13(0.01)L \quad \text{for } ^{209}\text{Bi}.$$

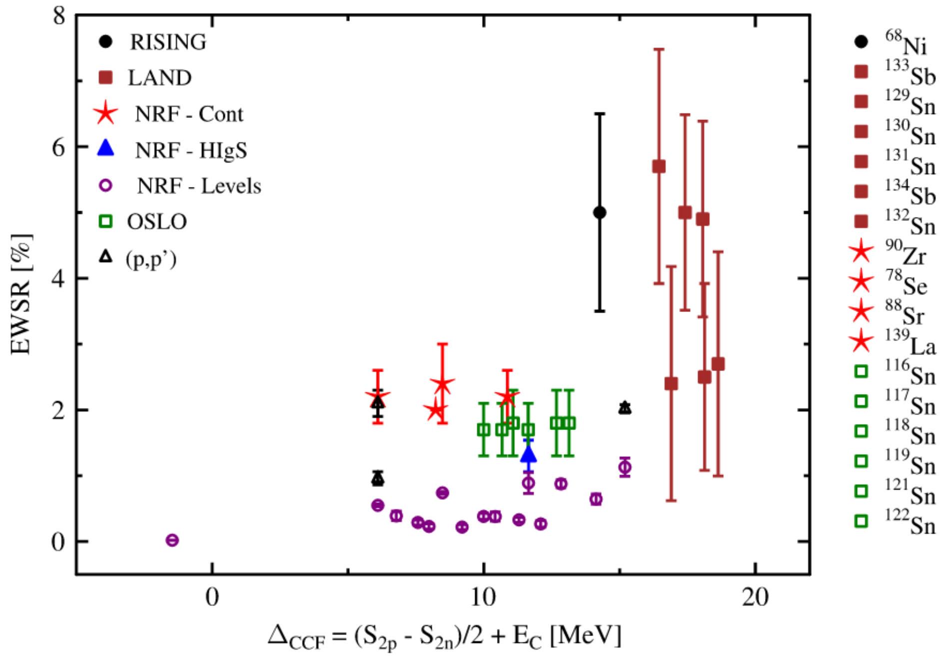


S. Bassauer, P. von Neumann-Cosel, P.-G. Reinhard et al. *Physics Letters B 810 (2020) 135804*

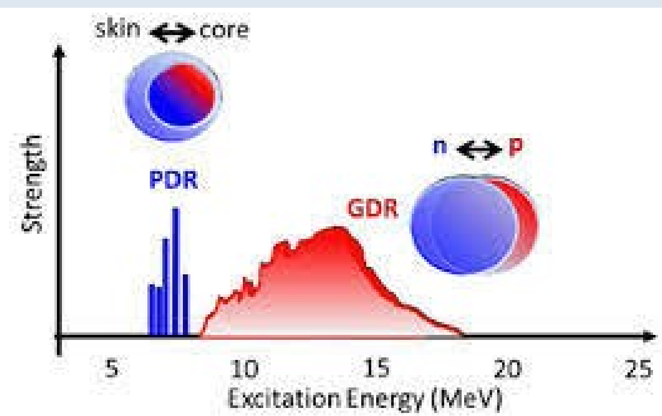
Pygmy dipole resonance / strenght (!?)

→ **Low energy strength** in the dipole (but also other) nuclear responses **impacts** the nuclear **polarizability** as well as **the r-process nucleosynthesis**





→ **No** clear **connection** with the nuclear **EoS** has been stablished **yet** (skin oscillation, toroidal mode, …)

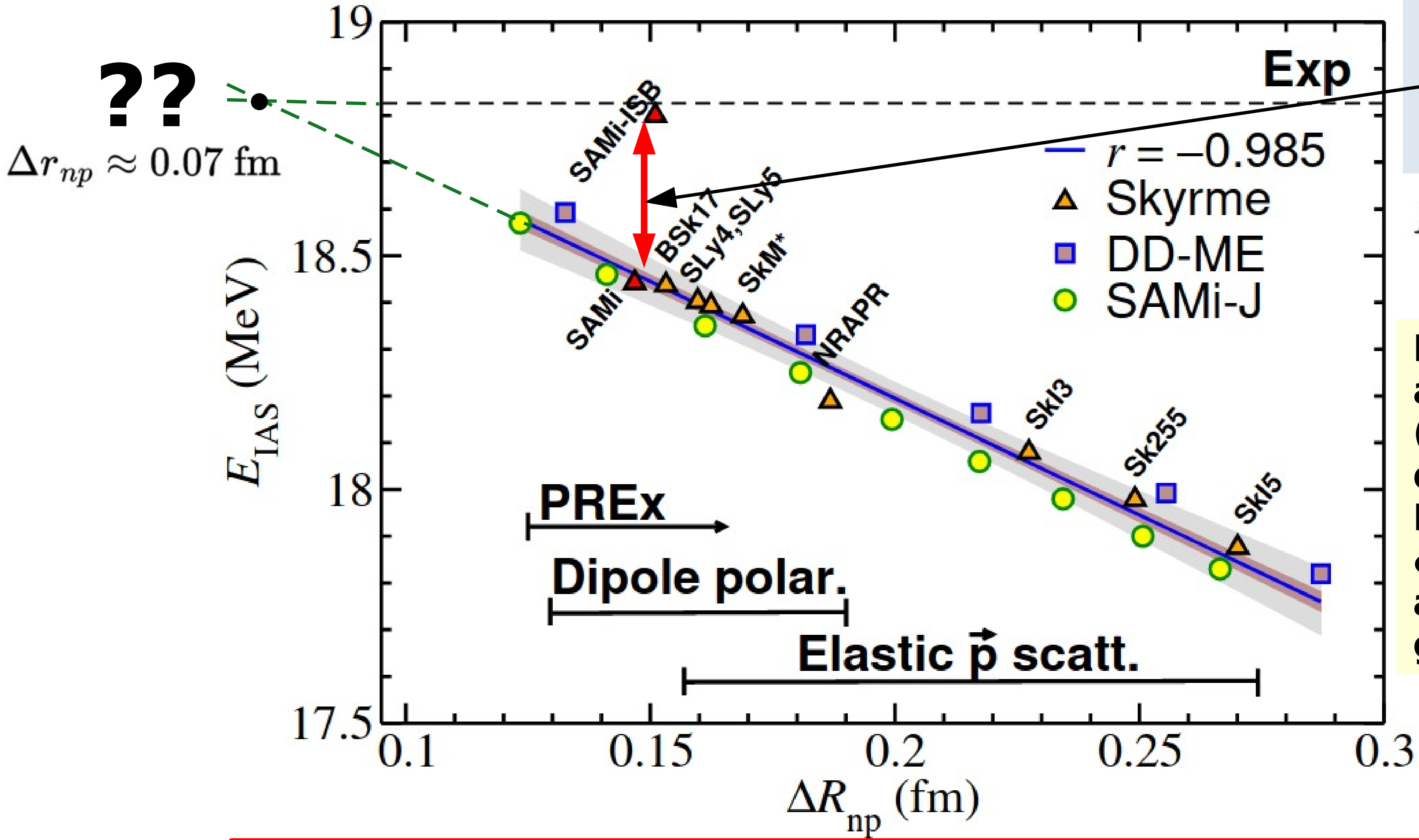


Progress in Particle and Nuclear Physics 70 (2013) 210–245

Progress in Particle and Nuclear Physics 106 (2019) 360–433

Charge exchange Giant Resonances: some examples

Isobaric Analog State



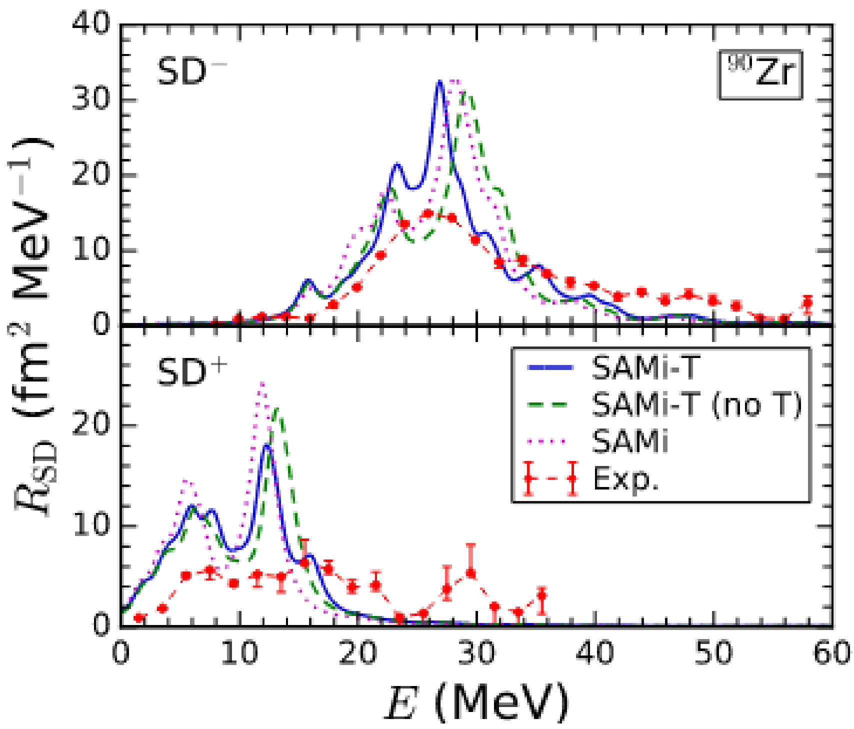
Isospin symmetry breaking in the nuclear force

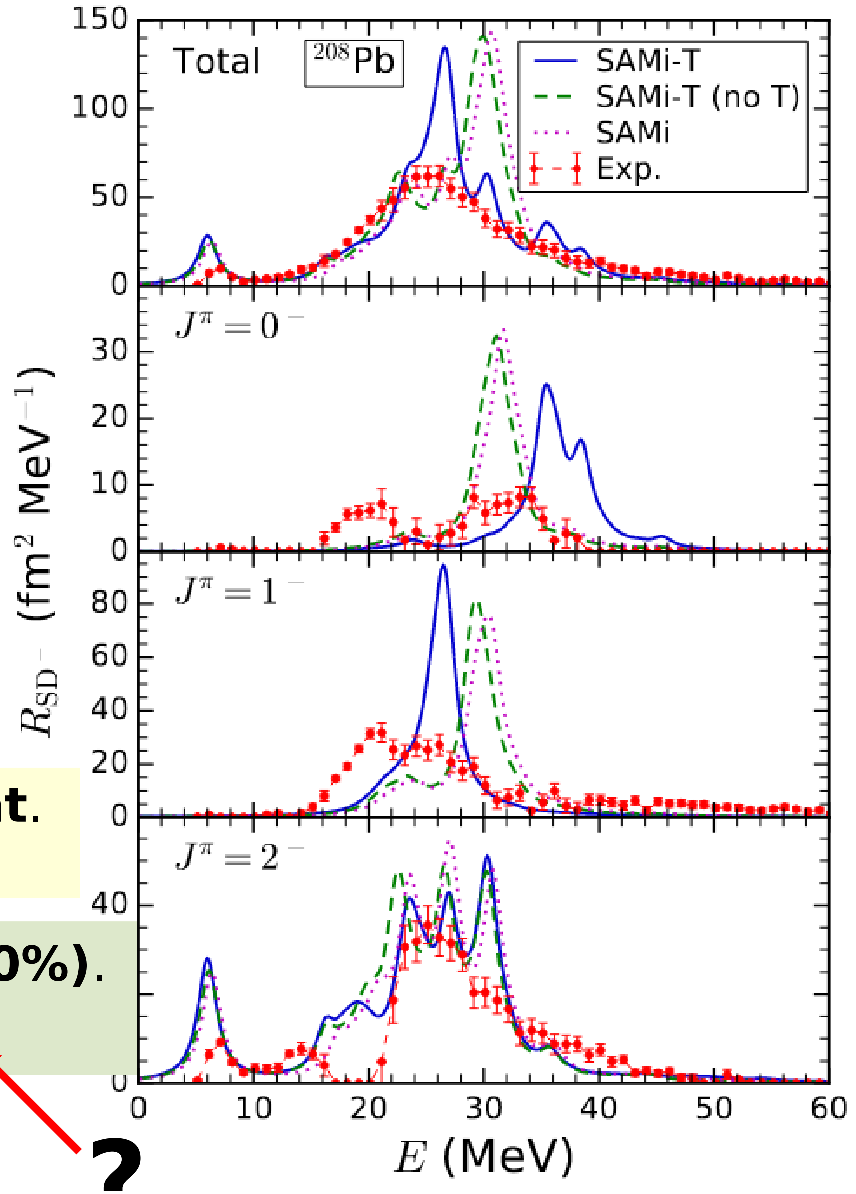
$E_{\mathrm{IAS}} \sim \langle T_+[\mathcal{H}, T_-]\rangle$

Further studies are needed (uncertainty quantification; Extended EDFs & ab initio guidance)

ISB, Δr_{np} and E_{IAS} → related among them

Spin Dipole Resonance





→ ^{90}Zr **exp and theo sum rules in agreement**.
From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.04$ fm

→ ^{208}Pb **exp sum rule < theo sum rule (~20%)**.
From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.03$ fm

?

Shihang Shen (申时行), Gianluca Colò, and Xavier Roca-Maza
Phys. Rev. C **99**, 034322 – Published 20 March 2019

Summary from Progress in Particle and Nuclear Physics 101 (2018) 96–176

K_0	$M(N,Z)$	220–245	Most accurate EDFs on $M(N,Z)$ and $\langle r^2_{ch}\rangle^{1/2}$ (see Section 5)
	ISGMR	220–260	From EDFs in closed shell nuclei [116]
	ISGMR	250–315	Blaizot's formula [Eq. (32)] [51]
	ISGMR	~200	EDF describing also open shell nuclei [118]
J	$M(N,Z)$	29-35.6	Most accurate EDFs on $M(N,Z)$ and $\langle r^2_{ch}\rangle^{1/2}$ (see Section 5)
	IVGDR	~24.1(8) + L/8	From EDF analysis [$S(\rho = 0.1\ \mathrm{fm}^{-3}) = 24.1(8)$ MeV] [273]
	PDS	30.2–33.8	From EDF analysis [370]
	PDS	31.0–33.6	From EDF analysis [371]
	α_D	24.5(8) + 0.168(7)L	From EDF analysis ^{208}Pb [96]
	α_D	30–35	From EDF analysis [179]
	IAS and Δr_{np}	30.2–33.7	From EDF analysis [325]
	AGDR	31.2–35.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	32–33	From EDF analysis [508]
	compilation	29.0–32.7	[106]
	compilation	30.7–32.5	[107]
	compilation	28.5–34.9	[3]
L	$M(N,Z)$	27–113	Most accurate EDFs on $M(N,Z)$ $\langle r^2_{ch}\rangle^{1/2}$ (see Section 5)
	ρ_n	40–110	proton-^{208}Pb scattering [24]
	ρ_n	0–60	π photoproduction (^{208}Pb) [181]
	ρ_n	30–80	antiprotonic at. (EDF analysis) [102,509]
	ρ_{weak}	>20	Parity violating scattering [27]
	PDS	32–54	From EDF analysis [370]
	PDS	49.1–80.5	From EDF analysis [371]
	α_D	20–66	From EDF analysis [179]
	IVGQR and ISGQR	19–55	From EDF analysis [101]
	IAS and Δr_{np}	35–75	From EDF analysis [325]
	AGDR	75.2–122.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	45.2–54.6	From EDF analysis [508]
	compilation	40.5–61.9	[106]
	compilation	42.4–75.4	[107]
	compilation	30.6–86.8	[3]
K_τ	ρ_n	−620 to −400	antiprotonic at. (EDF analysis) [102]
	ISGMR	−650 to −450	α-scattering Sn isotopes [55]
	ISGMR	−630 to −480	α-scattering Cd isotopes [111]
	ISGMR	−840 to −350	Blaizot's formula [Eq. (32)] [51]

200-260 MeV
25%

30-35 MeV
15%

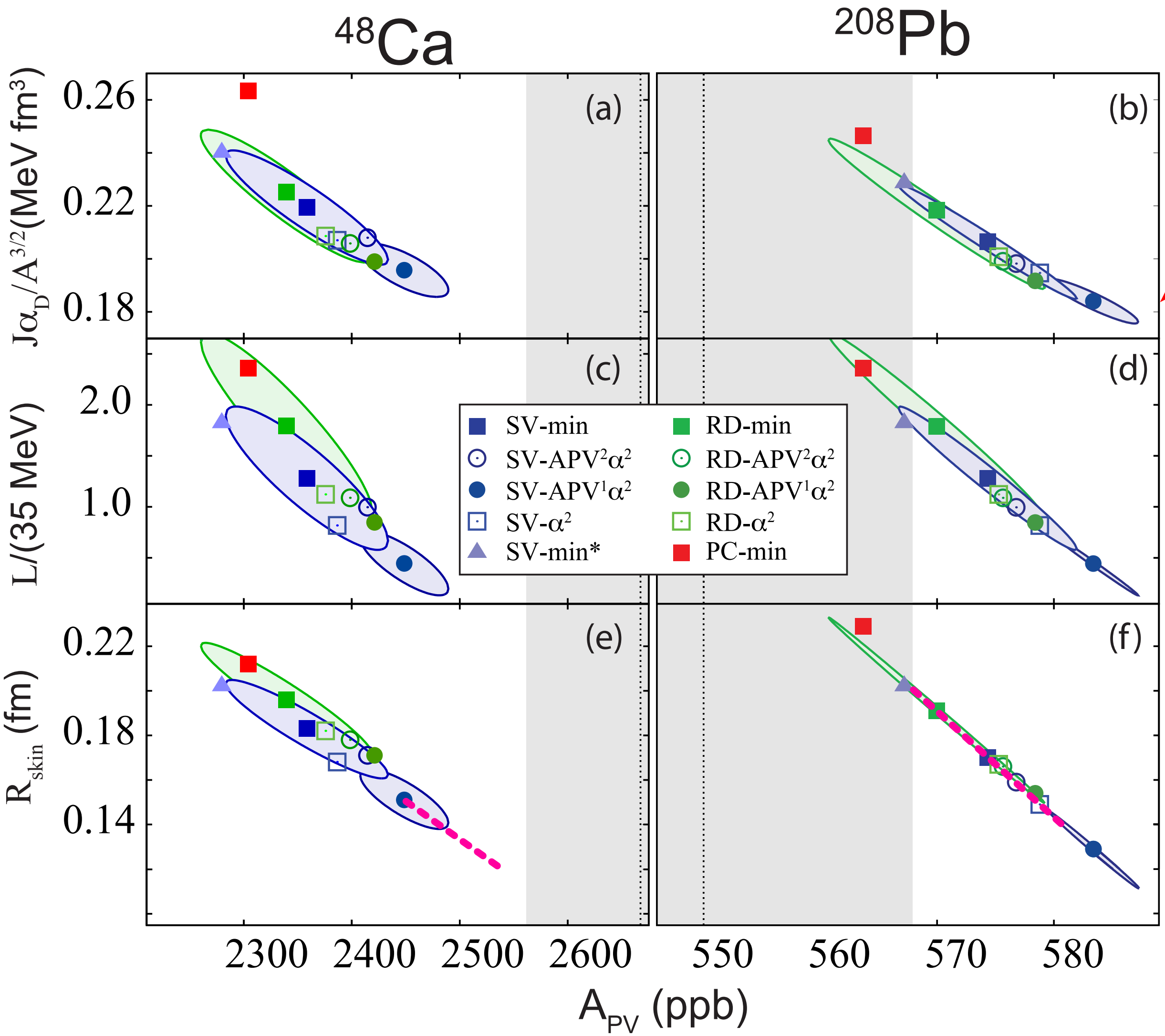
20-120 MeV
150%

??

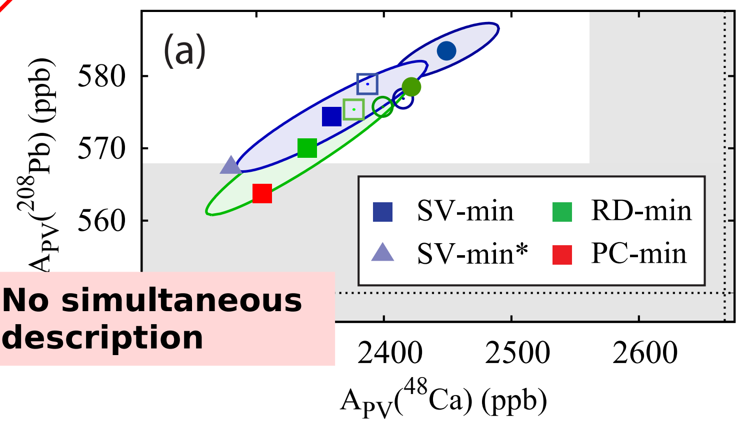
$\Delta\rho_0/\rho_0 \sim 2\%$; $\Delta e_0/e_0 \sim 4\%$

Blaizot's formula qualitative, not for exp. analysis
(see A. Phys. Pol. B Supp. 8 707, 2015)

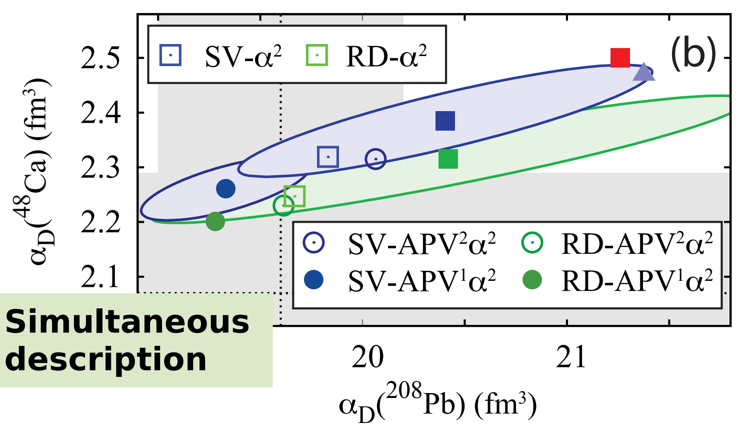
Parity Violating Electron scattering and Dipole polarizability: ^{48}Ca and ^{208}Pb



Theoretical (EDFs and ab initio) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca



No simultaneous description



Simultaneous description

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
arXiv:2206.03134 (2022)

Conclusions

→ The **isospin dependence** of the nuclear **EoS** around saturation density (mostly L) and, to a lesser extent, **the nuclear matter incompressibility** (K) remain to be accurately determined.

→ **Experimental and theoretical** efforts in finding and measuring observables specially **sensitive** to the **EoS** properties is of paramount **importance** for **low-energy nuclear physics** and **nuclear astrophysics.**

→ **PREx & CREx:** Until the **tension** between **theory and experiment** is **resolved,** one should exercise extreme **caution** when interpreting the new **A_{PV} measurements** in the context of **neutron skins** or **nuclear symmetry energy**

Collaborators

→ Gianluca **Colò** (University of Milan)

→ Hiroyuki **Sagawa** (University of Aizu & RIKEN)

→ Shihang **Shen** (Forschungszentrum Jülich)

→ Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)

→ Jorge **Piekarewicz** (Florida State University)

→ Nils **Paar** & Dario **Vretenar** (University of Zagreb)

→ Bijay K. **Agrawal** (Saha Institute of Nuclear Physics)

→ P.-G. **Reinhard** (University of Erlangen-Nürnberg)

→ Yifei **Niu** (Lanzhou University)

→ Witold **Nazarewicz** (FRIB and Michigan State University)

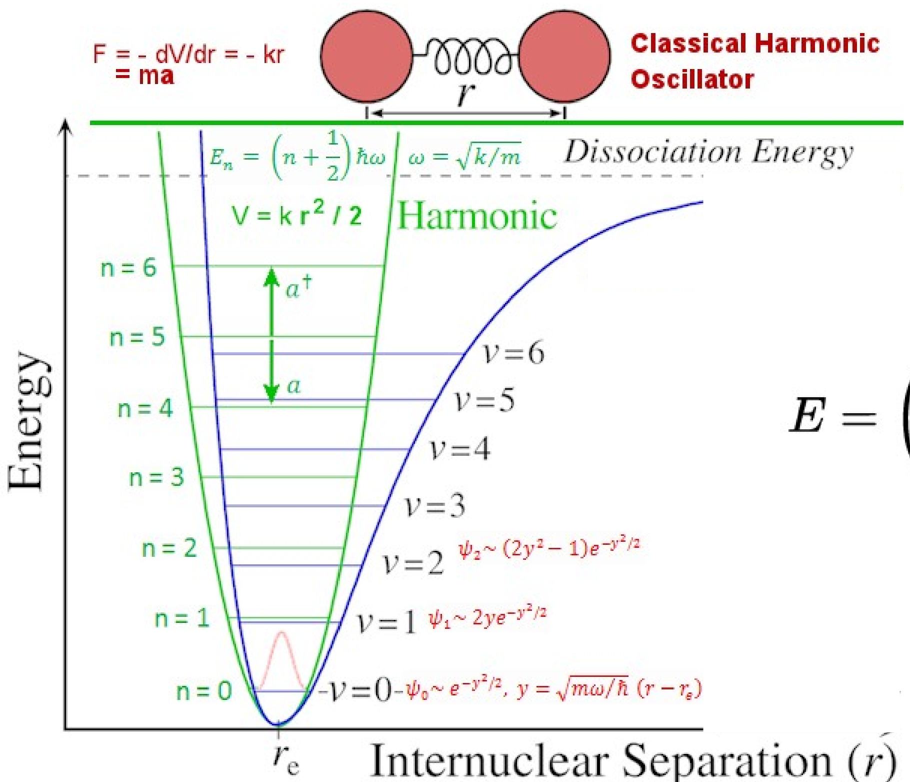
→ Stephane **Goriely** (Université Libre de Bruxelles)

→ Sophie **Péru** (Université Paris-Saclay, CEA)

Backup

Giant Resonances: Harmonic oscillator

Assuming **nucleons** as **non-interacting** fermions confined in an **Harmonic Oscillator** (HO) trap with suitable **hω₀** (shell gap) and apply **harmonic perturbation**



The **HO Hamiltonian**, in terms of the conjugate variables α and dα/dt and the C and B parameters under the action of an harmonic perturbation (**F**=-κ**α**), could be written as (B&M):

$$\mathcal{H} = \mathcal{H}_0 + \quad F \rightarrow \mathcal{H} = \frac{1}{2}B_0\dot{\alpha}^2 + \frac{1}{2}(C_0+\kappa)\alpha^2$$

$$E = \left(\frac{1}{B_0}\frac{\partial^2 U}{\partial \alpha^2}\right)^{1/2} \qquad E = E_0\left(1+\frac{\kappa}{C_0}\right)^{1/2}$$

Depending on the type of perturbation →

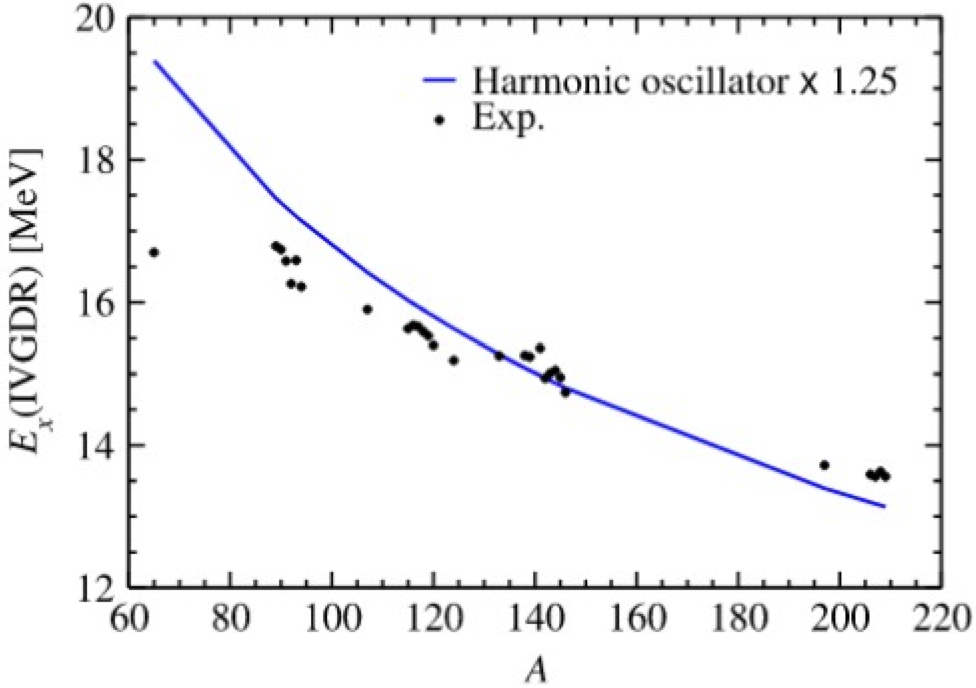
$$E \propto \left(\frac{\partial^2 e}{\partial \delta^2}\right)^{1/2} = [S(\rho)]^{1/2}$$

$$E \propto \left(\frac{\partial^2 e}{\partial \rho^2}\right)^{1/2} = [K]^{1/2}$$

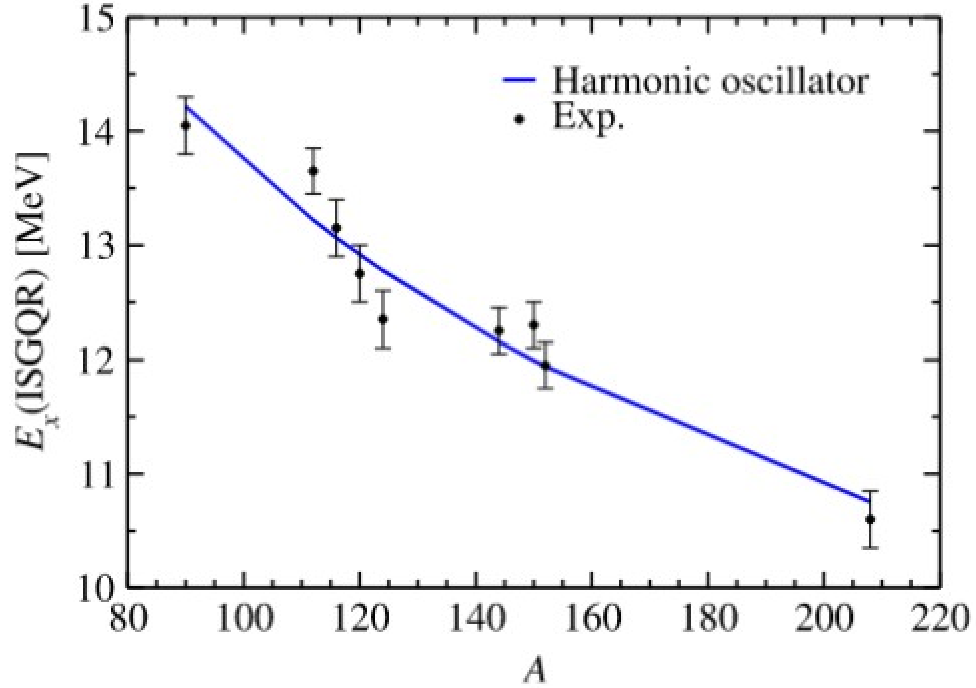
(Fig. modified from https://universe-review.ca/R15-33-harmonics.htm)

Giant Resonances: HO qualitative trends

IV Giant Dipole Resonance



IS Giant Quadrupole Resonance

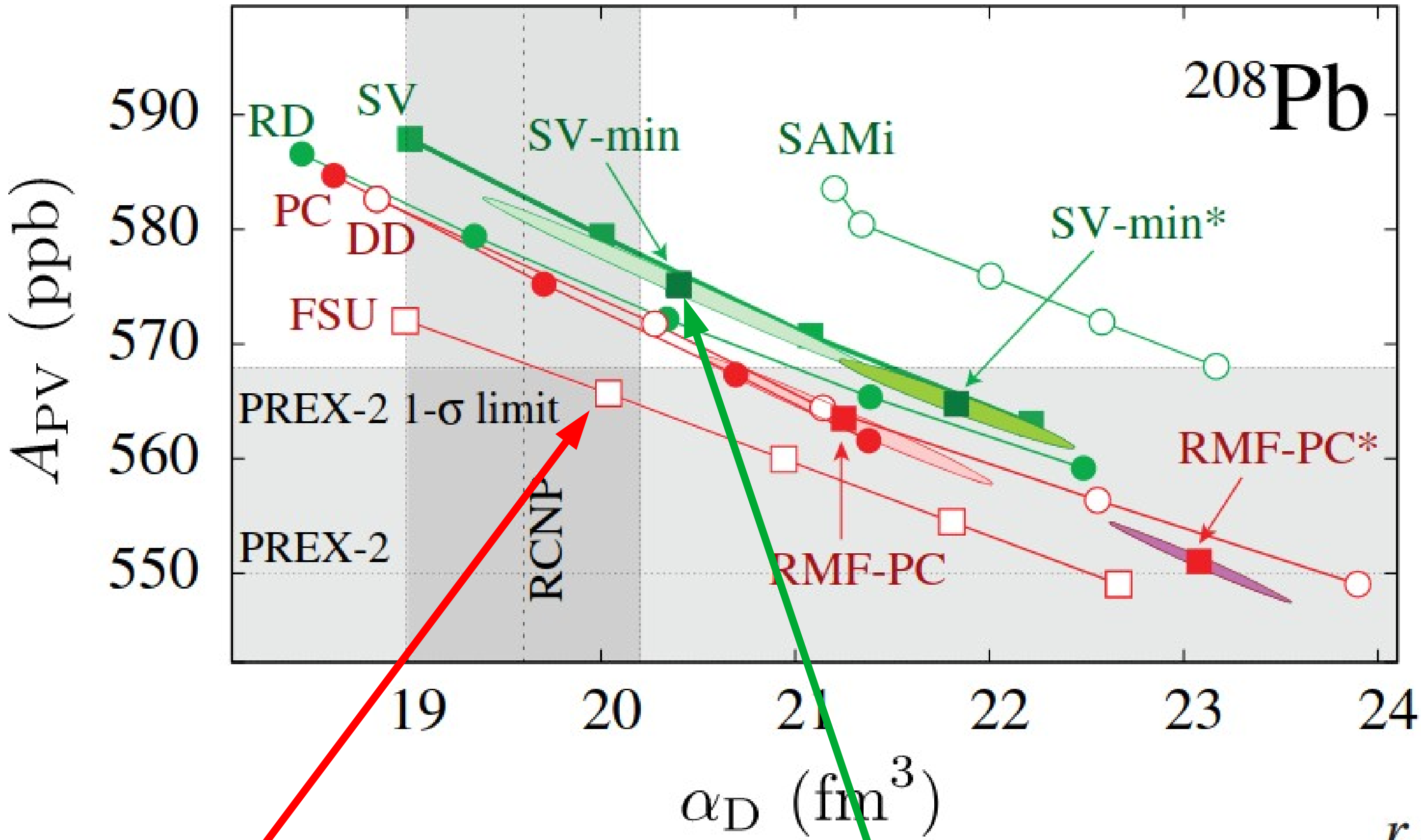


$$\hbar\omega^{\mathrm{IVGDR}} = \hbar\omega_0 \left(1 + \frac{\hbar^2}{4m\langle r^2\rangle} \frac{V_{\mathrm{sym}}}{\hbar^2\omega_0^2}\right)^{1/2}$$

$$\hbar\omega^{\mathrm{ISGQR}} = \sqrt{2\frac{m}{m^*}}\hbar\omega_0$$

X. Roca-Maza, N. Paar / Progress in Particle and Nuclear Physics 101 (2018) 96–176

Parity Violating Electron scattering and Dipole polarizability: ^{208}Pb



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. 127, 232501 (2021)

SV-min, suitable model to predict EoS around ρ_0 and Δr_{np}

$$r_{\text{skin}} = 0.19 \pm 0.02 \text{ fm}$$

$$L = 54 \pm 8 \text{ MeV}$$

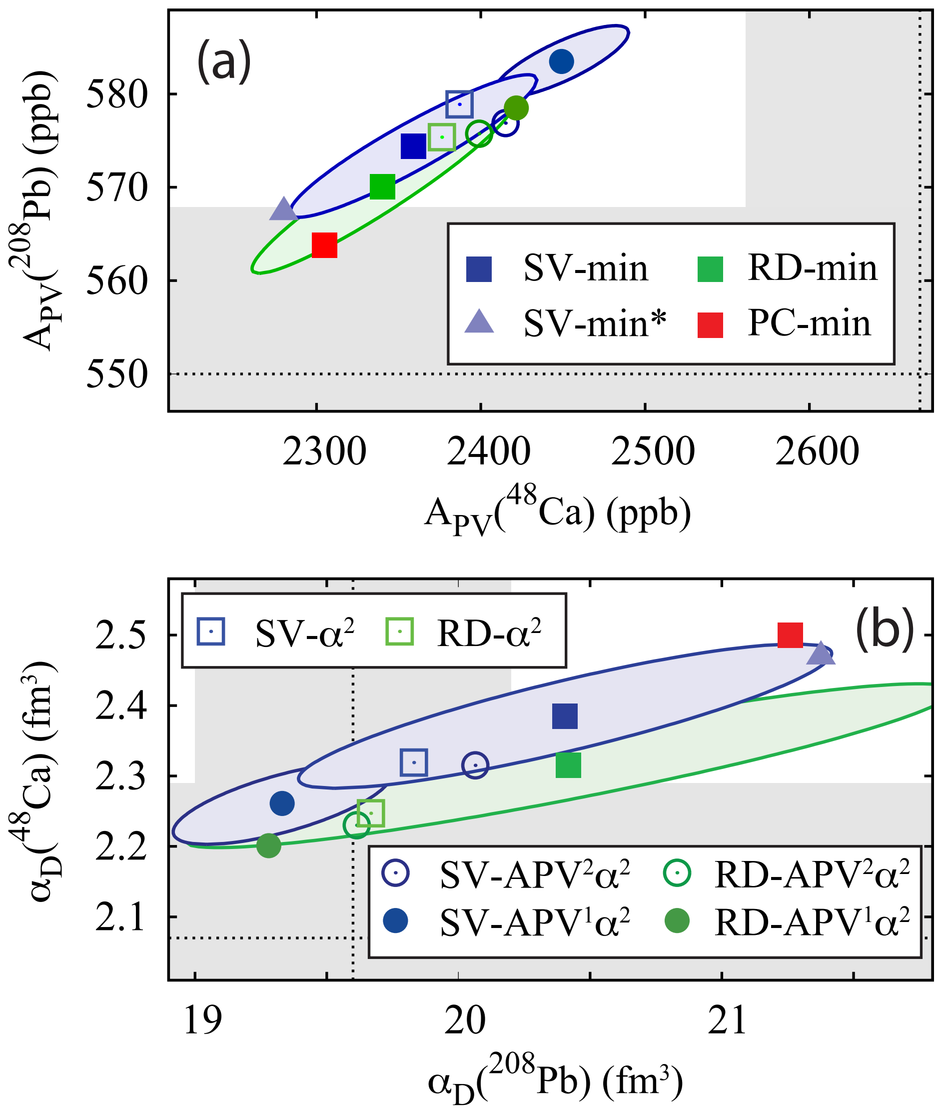
B and **Rch** away from "expected" EDF accuracy for B and Rch

Theoretical and experimental 1σ errors overlap for SV-min but not overlap both exp.

Parity Violating Electron scattering and Dipole polarizability: ^{48}Ca & ^{208}Pb

No simultaneous description

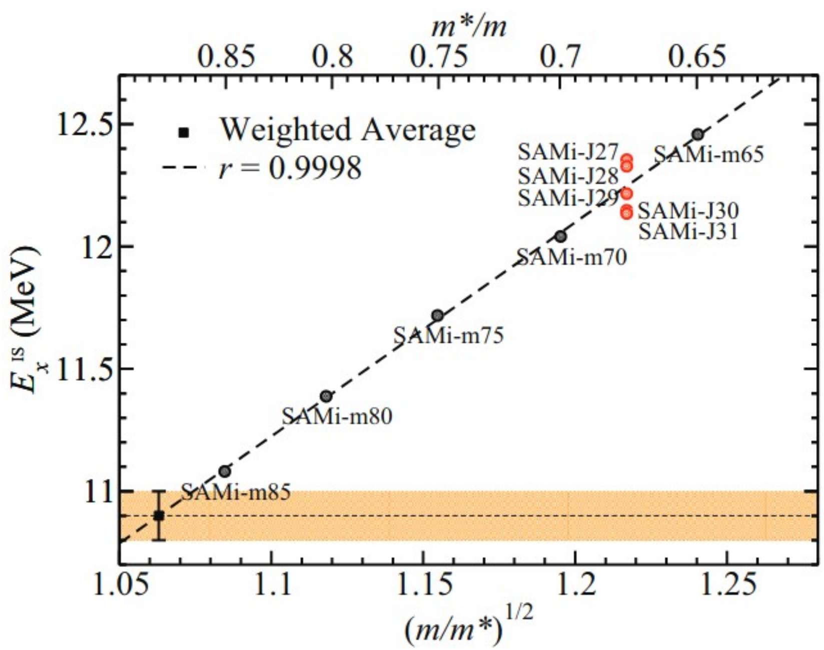
Simultaneous description



↔ **?**

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz arXiv:2206.03134 (2022)

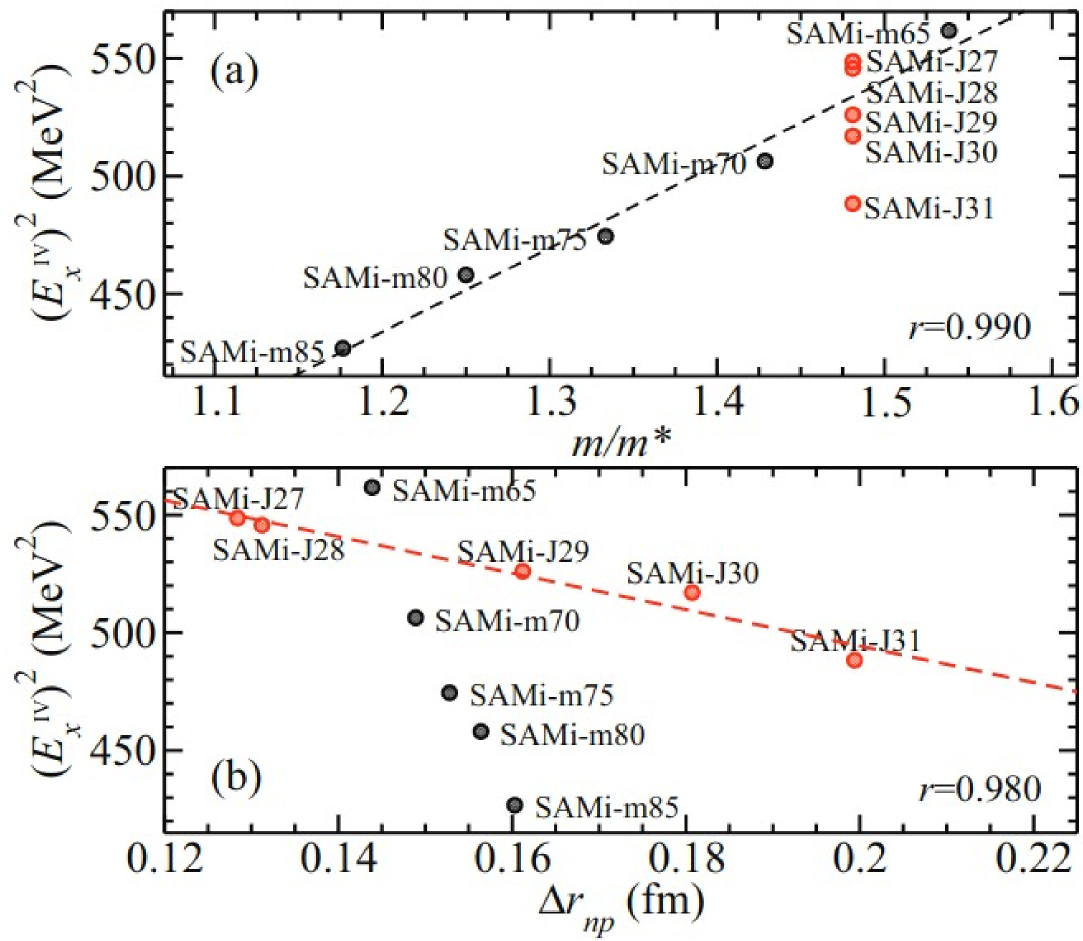
Giant Quadrupole resonance



$$E_x^{\rm IS} = \sqrt{2m/m^*}\hbar\omega_0,$$

From HO one would expect dependence on m* and symmetry energy:

$$E_x^{\rm IV} \approx 2\left[\frac{m}{m^*}(\hbar\omega_0)^2 + 2\frac{\varepsilon_{F_\infty}^2}{A^{2/3}}\left(\frac{3S(\rho_A)}{\varepsilon_{F_\infty}} - 1\right)\right]^{1/2}$$



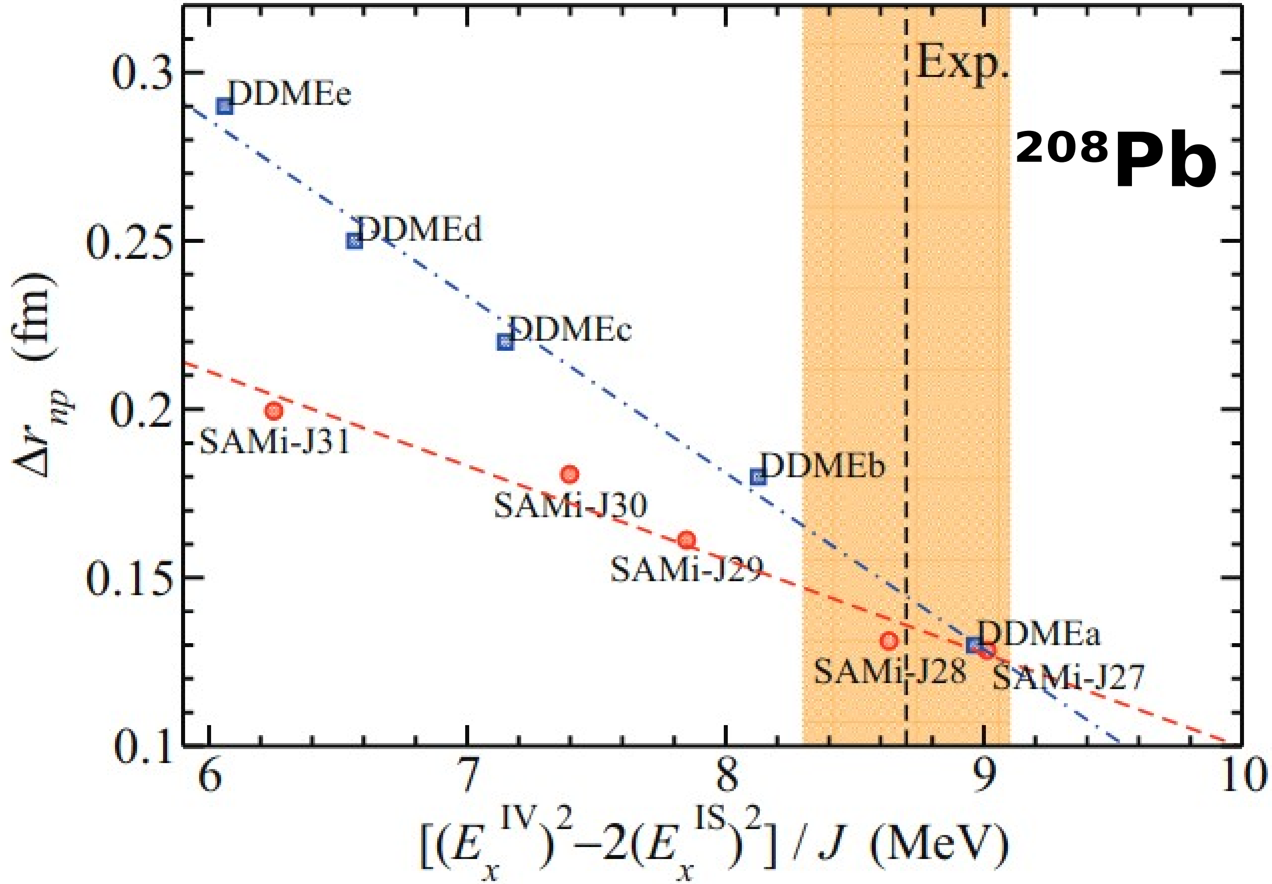
X. Roca-Maza, M. Brenna, B. K. Agrawal, P. F. Bortignon, G. Colò, Phys. Rev. C **87**, 034301 – Published 1 March 2013

Giant Quadrupole Resonance

→ **From HO model one would expect:**

$$\Delta r_{np} \sim \left[\frac{(E_x^{\mathrm{IV}})^2 - 2(E_x^{\mathrm{IS}})^2}{J} \right]$$

$$\frac{\Delta r_{np} - \Delta r_{np}^{\mathrm{surf}}}{\langle r^2 \rangle^{1/2}} = \frac{2}{3}(I - I_C)\left\{1 - \frac{\varepsilon_{F_\infty}}{3J} - \frac{3}{7}\frac{I_C}{I - I_C} - \frac{A^{2/3}}{24\varepsilon_{F_\infty}}\left[\frac{(E_x^{\mathrm{IV}})^2 - 2(E_x^{\mathrm{IS}})^2}{J}\right]\right\}.$$



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Phys. Rev. C **87**, 034301 – Published 1 March 2013