Equation of State for neutron star merger simulations

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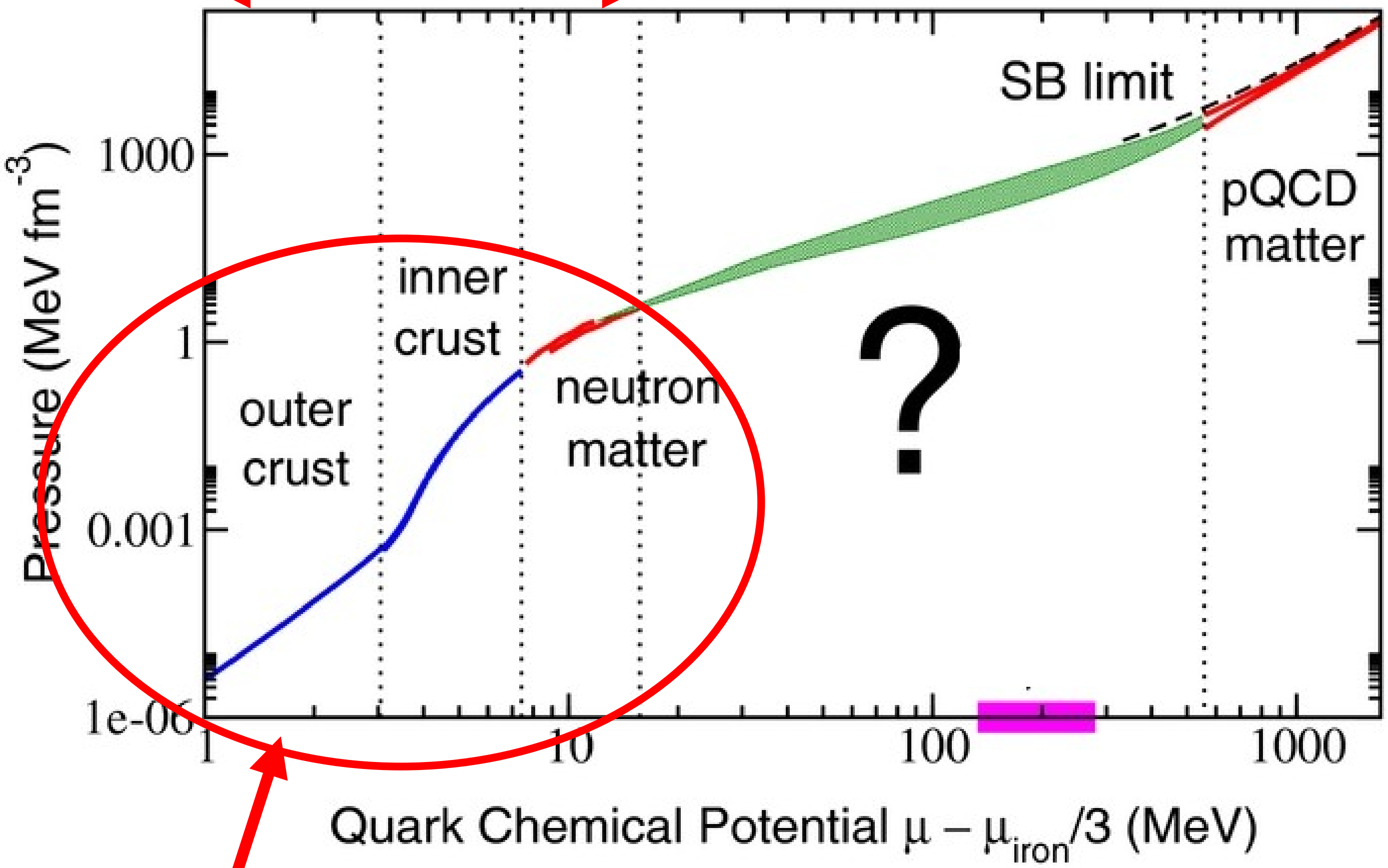
Neutron Star Equation of State

$\rho_0 = 0.16\ \mathrm{fm}^{-3}$

$\rho \lesssim 2\rho_0$

$2\rho_0 \lesssim \rho \lesssim 8\rho_0$

$\rho_{\mathrm{pQCD}} \gtrsim 40\rho_0$



Low energy Nuclear Physics Domain

CONSTRAINING NEUTRON STAR MATTER WITH QUANTUM CHROMODYNAMICS

Aleksi Kurkela[1], Eduardo S. Fraga[2,3,4], Jürgen Schaffner-Bielich[2], and Aleksi Vuorinen[5]

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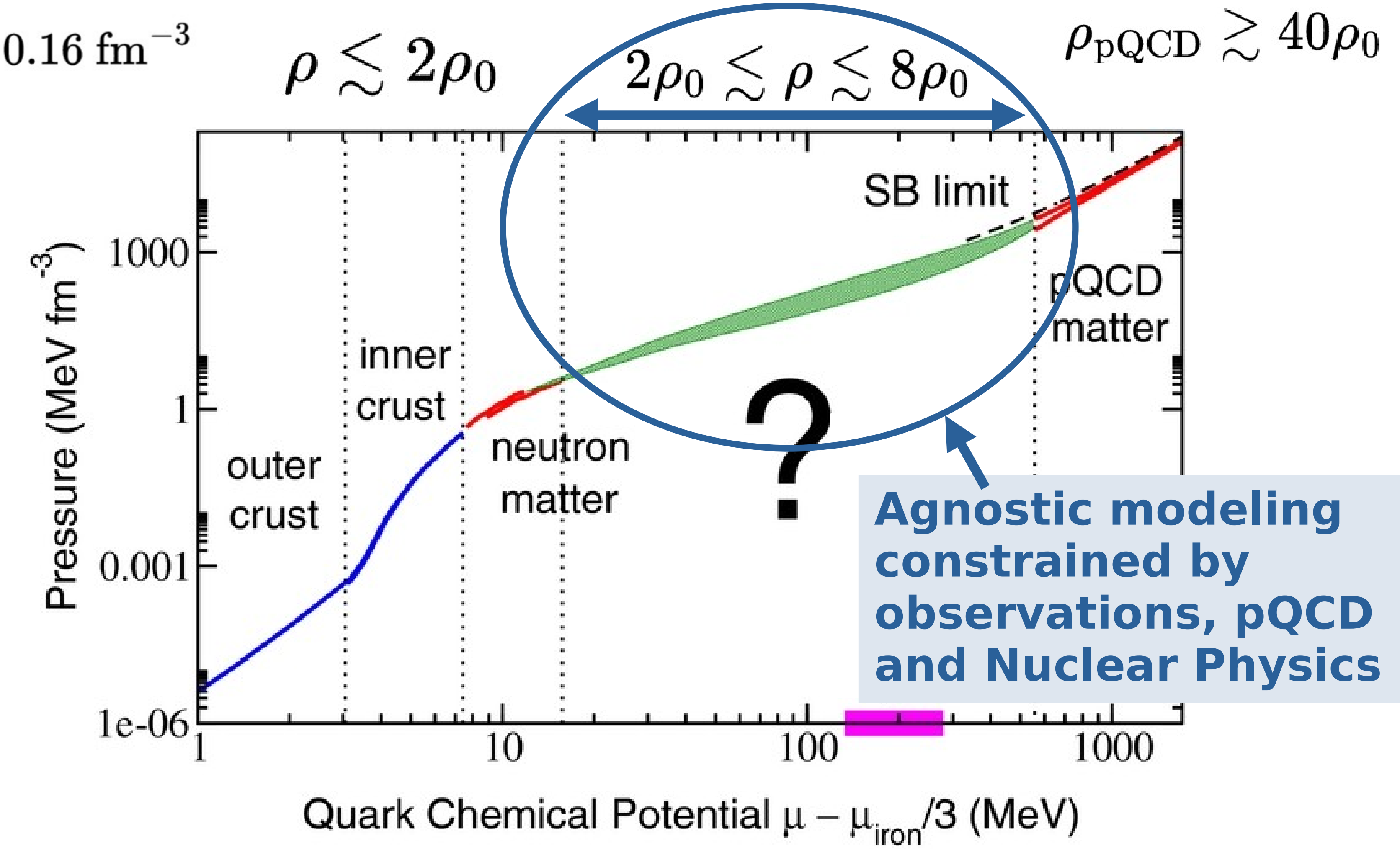
The Astrophysical Journal, Volume 789, Number 2
Citation Aleksi Kurkela *et al* 2014 *ApJ* **789** 127

Neutron Star Equation of State



CONSTRAINING NEUTRON STAR MATTER WITH QUANTUM CHROMODYNAMICS

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Nuclear Equation of State (EoS)

Main ingredient to build the neutron star EoS at low densities < 1-2ρ_0

Definition: the **energy** per **nucleon** (e=E/A where A=N+Z) of an **uniform system of neutrons and protons** as a function of the **neutron** (ρ_n = N/V) and **proton** (ρ_p = Z/V) **densities**, at **zero temperature**, **unpolarized**, assuming **isospin symmetry** and **neglecting Coulomb** effects among protons.

Why???

→ **Zero temperature:** room temperature 10^2K→$\mathbf{10^{-8}}$ **MeV** while "cold" neutron stars are at about 10^9K→0.1 MeV. **Separation energy** in stable nuclei (equivalent to ionization energy in atoms) is of **several MeV**.

→ **Unpolarized:** energy favours **couples** of neutrons and protons **occupying the same state but with opposite spins** (equivalent to electrons in atoms)

→ **Isospin symmetry:** neutron-neutron, proton-proton and neutron-proton **nuclear interaction** are very **similar** among them. **Masses** of neutrons and protons are almost **degenerate**. Hence neutrons and protons can be thought as **two states** of the **same particle** with different isobaric spin or **isospin** (in analogy with spin): the **nucleon.**

→ **No Coulomb:** **idealized uniform system (focus on strong interaction)**. Real systems are finite and frequently electrically neutral so no problems (divergences) in adding Coulomb.

[Besides that, the strong interaction at the typical scale of a nucleus is much stronger than the Coulomb interaction and the Coulomb energy (*) per particle of an infinite system of protons would be infinite.]

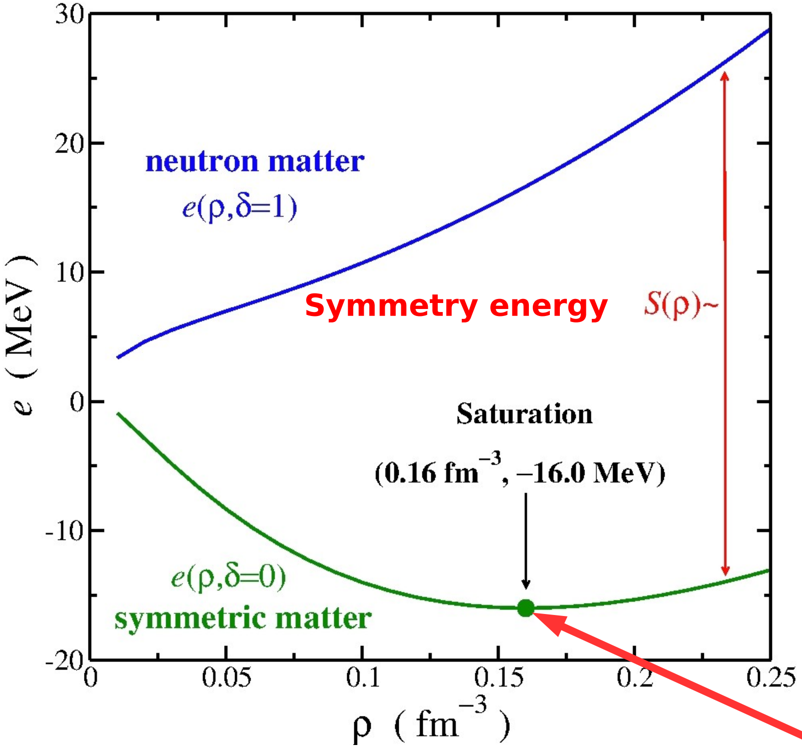
$$(*)\text{Coulomb interaction infinite range!!} \rightarrow \frac{E_C}{A} \sim \frac{Z(Z-1)}{A^{4/3}} \sim Z^{2/3}$$

Nuclear Equation of State (EoS)

It is **convenient** to write the **energy per nucleon** (e) as a **function** of the **total density** [$\rho=\rho_n+\rho_p$] and their **relative difference** [$\delta=(\rho_n - \rho_p)/\rho$].

- → Due to isospin symmetry only even powers of δ will appear
- → Stable nuclei tend to show small values of δ

Taylor expansion for δ→ 0: $\boxed{e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4]}$



It is customary to also **expand** e(ρ,0) and S(ρ) around nuclear **saturation density** $\rho_0 \sim 0.16\ \mathrm{fm}^{-3}$

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0 x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\mathrm{sym}}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

K_0 → how **compressible** is symmetric matter at ρ_0

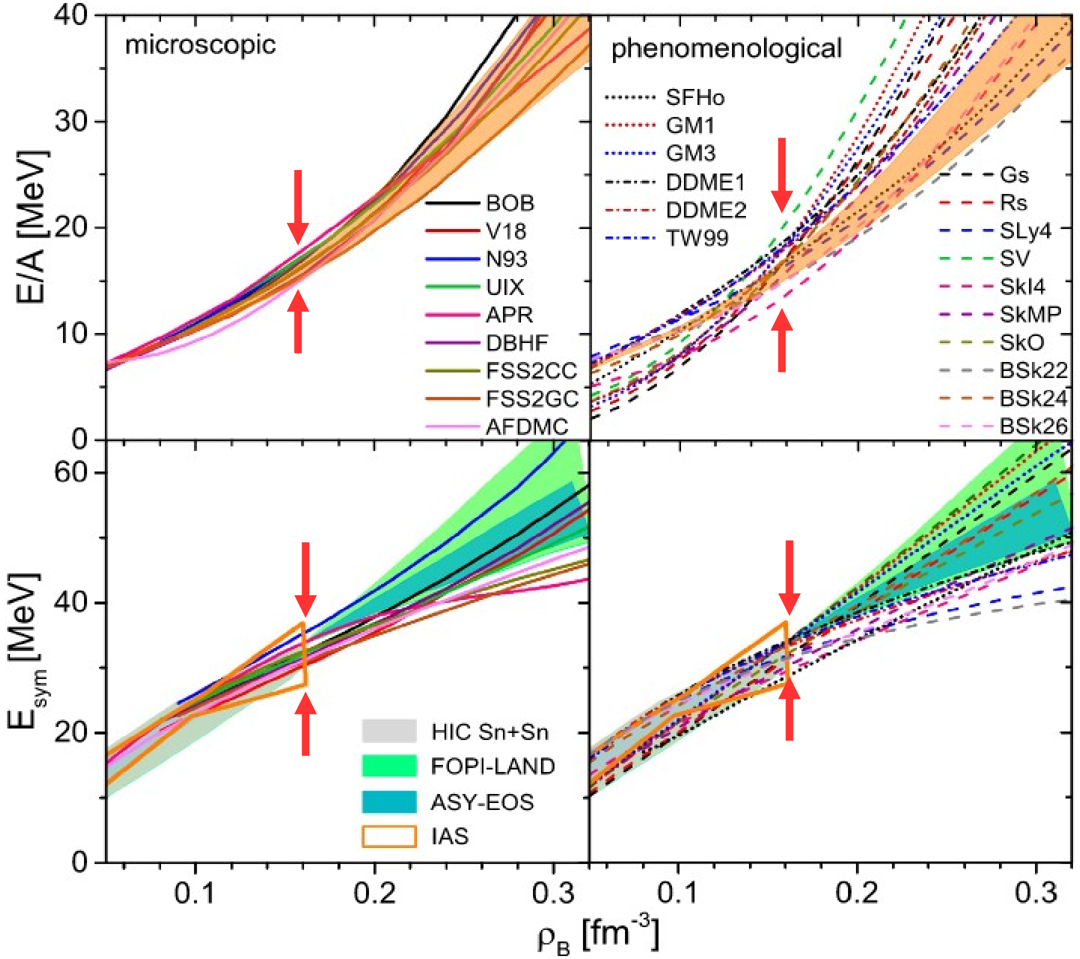
J (a_A) → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L (a_S) → **neutron pressure** in neutron matter at ρ_0

$$P(\rho=\rho_0,\delta=0) = 0\ \ \mathrm{MeV\ fm}^{-3}$$

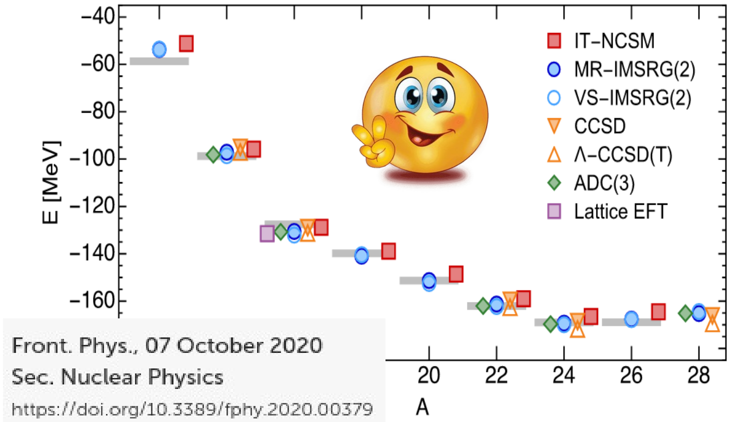
EoS from current nuclear models

Micorscopic and phenomenological models constrainted by different data display similar discrepances on the EoS

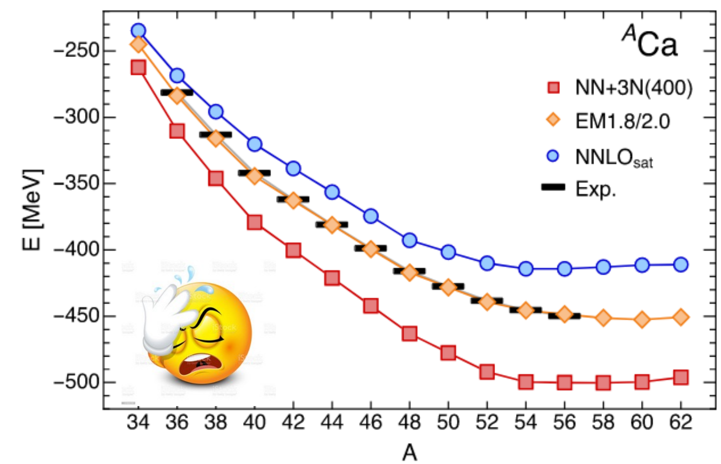


Symmetry energy not well constrained and specially **important** for the description of **dense neutron matter**

Many-body methods have been shown to agree



Main source of uncertainty in the nuclear Hamiltonian



Progress in Particle and Nuclear Physics 120 (2021) 103879

Light orange bands from χEFT from Drischler C., et al. Phys. Rev. Lett., 125 202702 (2020).

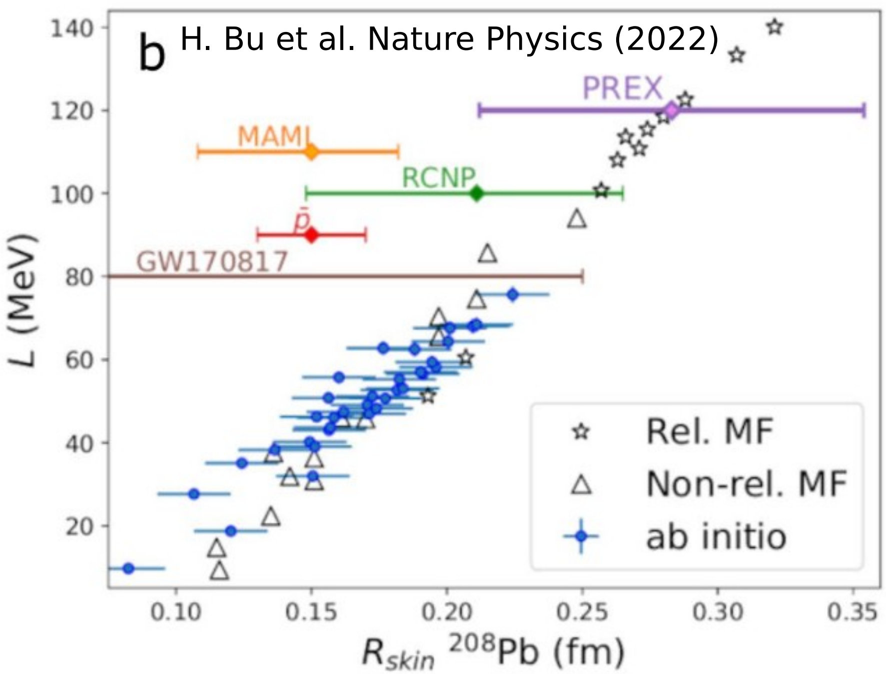
What can we learn from laboratory observables on the symmetry energy around ρ_0?

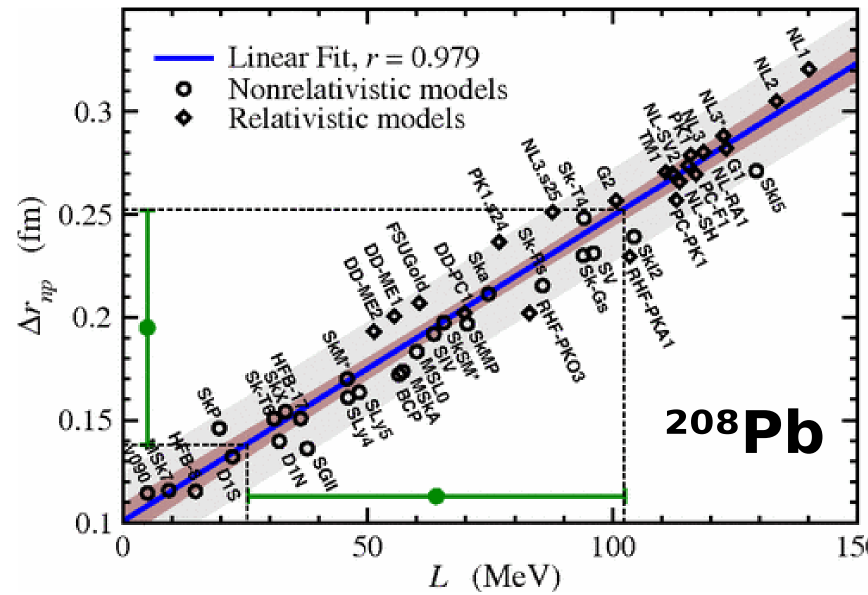
Neutron skin is a good proxy to L

B. Alex Brown Phys. Rev. Lett. 85, 5296 (2000)

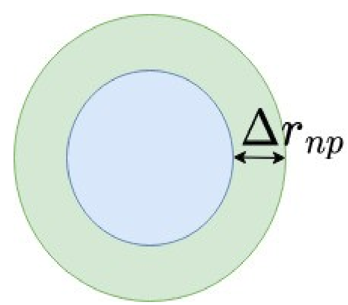
The **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in a heavy neutron rich nucleus is related to the **neutron pressure** ($\delta=1$) around ρ_0 (L).

$$P(\rho_0,\delta) = \rho_0^2 \frac{\partial e(\rho,\delta)}{\partial \rho}\bigg|_{\rho=\rho_0} = \frac{1}{3}\rho_0 \delta^2 L$$





Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



→ From the Droplet Model:

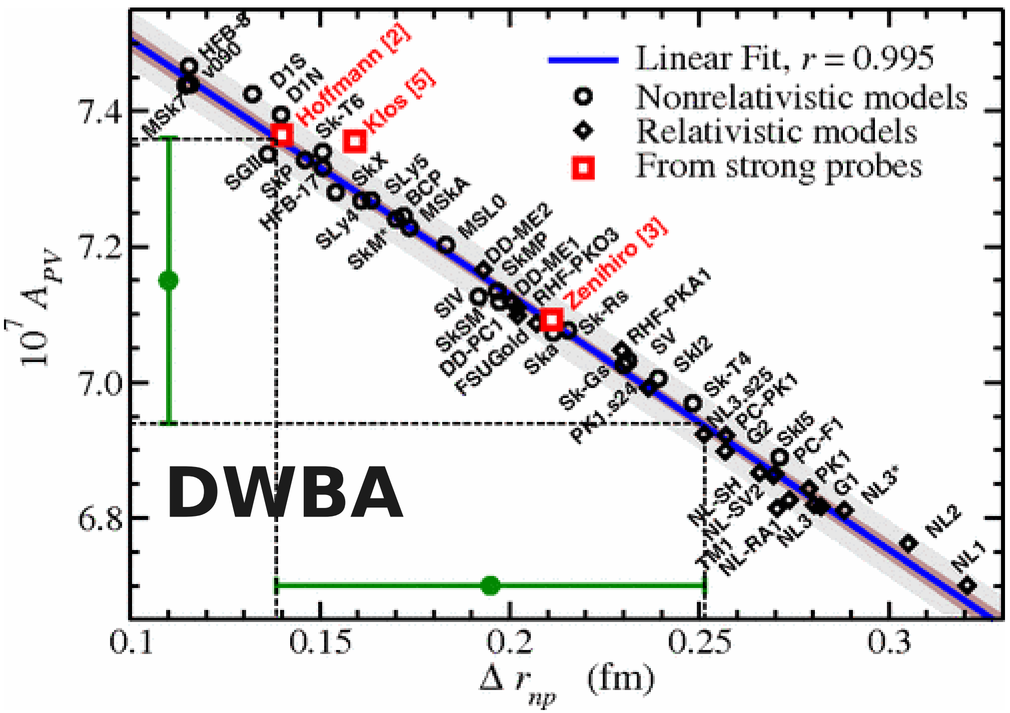
$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

But how can we measure the neutron skin thickness?

Polarized electron-Nucleus scattering:

→ In good approximation, **the weak interaction** probes the **neutron distribution** in nuclei while **Coulomb interaction** probes the **proton distribution**

→ **Different experimental efforts @ Jlab (USA) & MAMI (Germany)**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

→ **Electrons** interact by **exchanging** a **γ** (couples to **p**) or a **Z_0** boson (couples to **n**)

→ **Ultra-relativistic electrons**,depending on their helicity (±), will interact with the nucleus seeing a slightly different potential: **Coulomb** ± **Weak**

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

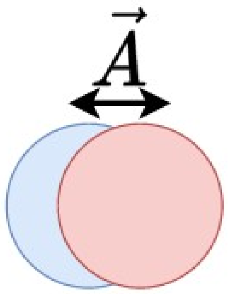
→ Main **unknown** is **ρ_n**

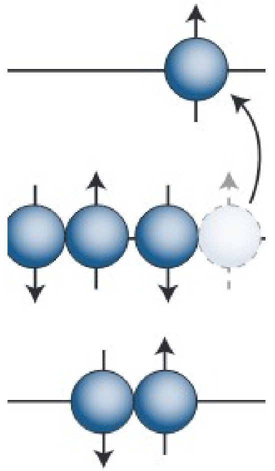
→ In **PWBA** for small momentum transfer **q**:

$$A_{pv} = \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 r_p^2}{3F_p(q)}\right)\Delta r_{np}$$

What else?
Giant Resonances

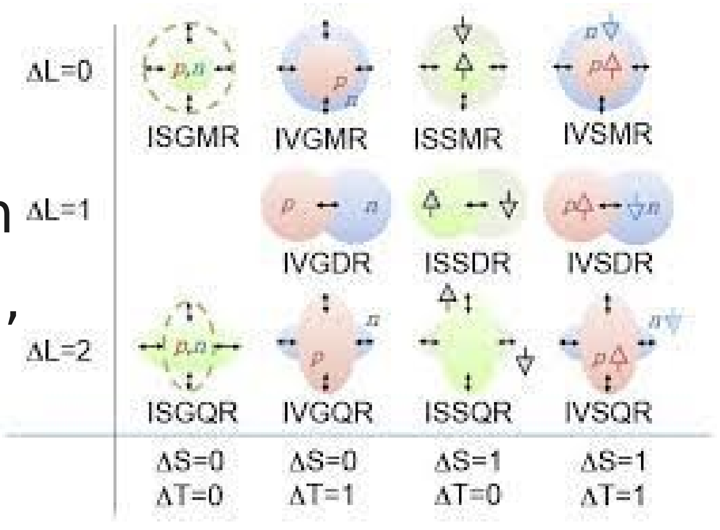
→ **Collective motion** giving rise to **Giant Resonances** could be **qualitatively** understood as *macroscopic* **oscillation/vibration** of the **nucleon densities** $\delta\rho \sim \mathbf{A}\cdot\nabla\rho$





→ **Microscopically,** overall Giant Resonance properties have been seen to be well described as a **coherent superposition** of **1 particle – 1 hole** excitations.

→ **Collective excitations** can be **classified** in terms of the quantum numbers **J = L + S** and **parity**, $\pi=(-1)^L$, and modeled by using transition operators of the type $\sum \tau_a r^L (\mathbf{Y}_{LM} \times \boldsymbol{\sigma})_{JM.}$ with $\tau_a=1$, τ_z for non-charge exchange IS and IV, respectively, or $\tau_a=\tau_{\pm}$ for charge exchange.



Giant Resonances: some guidance

Non interacting nucleons confined in an harmonic oscillator potential and perturbed with an harmonic perturbation

The **HO Hamiltonian**, in terms of the conjugate variables α (or r) and dα/dt (or v) and the ($C_0 \leftrightarrow m\omega^2$) and ($B_0 \leftrightarrow m$) parameters, could be written as (Bohr&Mottelson):

$$\mathcal{H}_0 = \frac{1}{2}B_0\dot{\alpha}^2 + \frac{1}{2}C_0\alpha^2 \rightarrow E_0 = \hbar\left(\frac{C_0}{B_0}\right)^{1/2}$$

$$E = \left(\frac{1}{B_0}\frac{\partial^2 U}{\partial \alpha^2}\right)^{1/2}$$

Assume harmonic perturbation (restoring force)

$$F = -\kappa\alpha \rightarrow V = \frac{1}{2}\kappa\alpha^2$$

$$\mathcal{H} = \mathcal{H}_0 + \ V \ \rightarrow \mathcal{H} = \frac{1}{2}B_0\dot{\alpha}^2 + \frac{1}{2}(C_0+\kappa)\alpha^2$$

$$E = E_0\left(1 + \frac{\kappa}{C_0}\right)^{1/2}$$

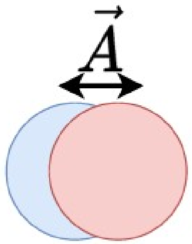
Restoring force

Depending on the type of perturbation

$$E \propto \left(\frac{\partial^2 e}{\partial \delta^2}\right)^{1/2} = [S(\rho)]^{1/2}$$

$$E \propto \left(\frac{\partial^2 e}{\partial \rho^2}\right)^{1/2} = [K]^{1/2}$$

Giant resonances: the IVGDR



→ The **Isovector Giant Dipole Resonance** was the first resonance measured (**photo-absorption** experiments)

→ The **cross section** for the **excitation** of the nucleus to a **final state** $|\nu\rangle$ with energy $\mathbf{E_\nu}$ from the **ground state** $|0\rangle$ with energy $\mathbf{E_0}$ by a **photon** at a given energy **E** can be written as

$$\sigma_\nu(E) = 4\pi^2\alpha(E_\nu - E_0)|\langle\nu|F_{\text{dipole}}|0\rangle|^2\delta(E - E_\nu + E_0)$$

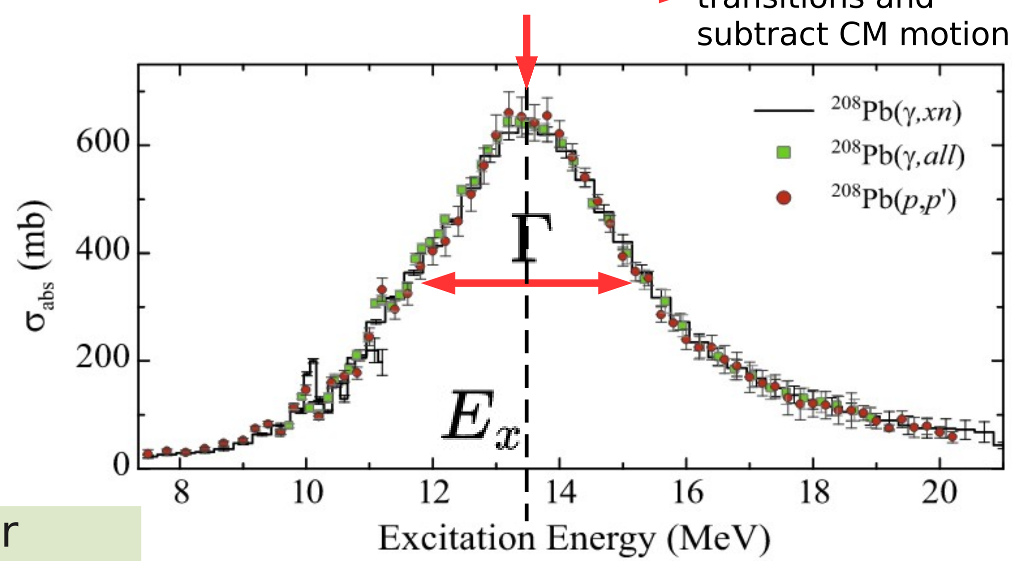
Convenient operator [~ r $Y_{10}(\Omega)$] produces dipole transitions and subtract CM motion

→ The **total cross-section** will be

$$\sigma_{\gamma-\text{abs}} = 4\pi^2\alpha\sum_\nu(E_\nu - E_0)|\langle\nu|F_{\text{dipole}}|0\rangle|^2$$

$$S(E) \equiv \sum_\nu |\langle\nu|F|0\rangle|^2\delta(E - E_\nu + E_0)$$

where **S(E)** is the so called **Strength function**

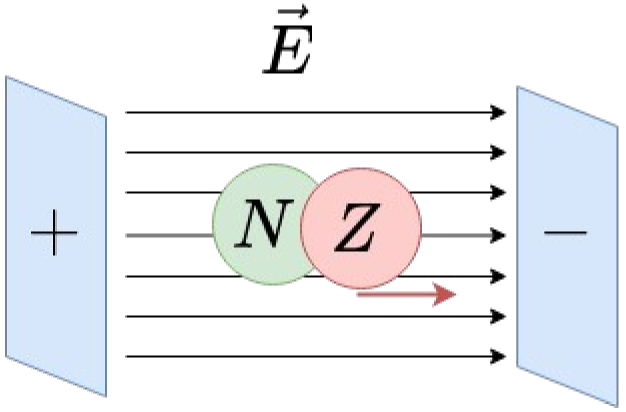


S(E) is used to **characterize** the nuclear **response** (experimentally, it is commonly parametrized as a Lorentzian function with energy E_x and width Γ)

Dipole polarizability

The **electric polarizability** measures **tendency** of the nuclear **charge distribution** to be **distorted**

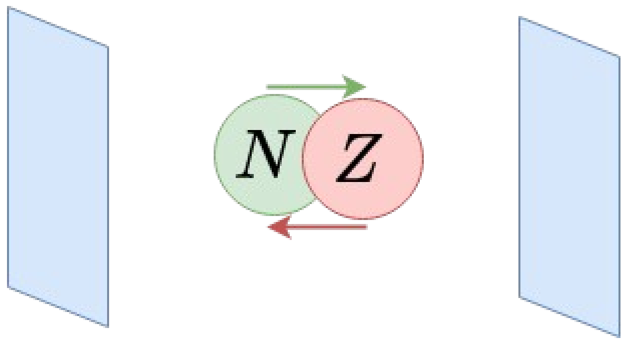
$$\alpha = \frac{\text{electric dipole moment}}{\text{external electric field applied}}$$



Polarizability is **proportional** to the photoabsorption cross section (see previous slide):

$$\alpha_D = \frac{\hbar c}{2\pi^2 e^2} \int \frac{\sigma_{\text{abs}}}{\omega^2} d\omega,$$

How easy is to separate neutrons from protons?
Symmetry energy will tell. Remember HO (slide 15).

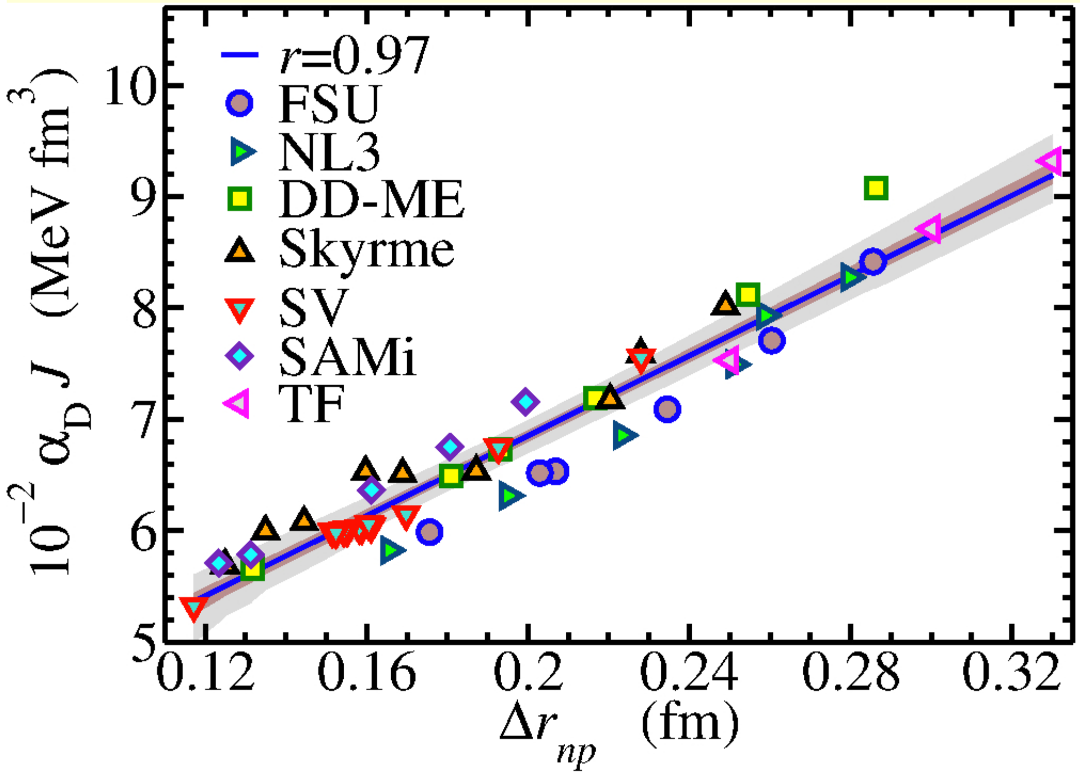


$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2$$

$$E_x \sim \sqrt{\frac{\partial^2 e(\rho, \delta)}{\partial \delta^2}} \sim \sqrt{S(\rho)}$$

Dipole polarizability and neutron skin

From **<u>state-of-the-art</u> nuclear models** one finds a **correlation** between **α_D**, **the symmetry energy** (J) and the **neutron skin thickness**



Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz

Phys. Rev. C 88, 024316 (2013)

→ This relation is also supported by simple models.

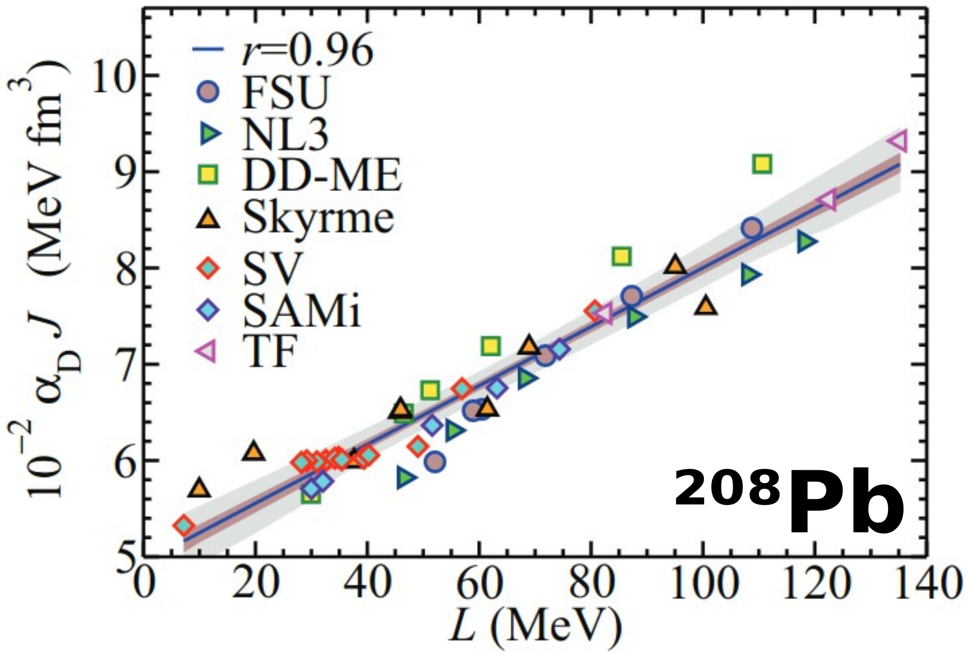
→ For guidance assuming the **Droplet Model**, one would find:

$$\alpha_D \approx \frac{\pi e^2}{54} \frac{\langle r^2 \rangle}{J} A \left(1 + \frac{5}{2} \frac{\Delta r_{np} - \Delta r_{np}^{\mathrm{surf}} - \Delta r_{np}^{\mathrm{Coul}}}{\langle r^2 \rangle^{1/2} (I - I_{\mathrm{Coul}})} \right)$$

Polarizability increases with the mass (for the <u>dipole $A^{5/3}$</u>, for the <u>quadrupole $A^{7/3}$</u> and so on …) and it **sets a relation between the <u>EoS parameters J and L</u>**

Dipole polarizability, J and L

Determination of the J vs L relation from experimental data according to EDFs



$$J = 25.0(2) + 0.19(2)L \quad \text{for } ^{68}\text{Ni},$$
$$J = 25.4(1.1) + 0.17(1)L \quad \text{for } ^{120}\text{Sn},$$
$$J = 24.5(8) + 0.17(1)L \quad \text{for } ^{208}\text{Pb}.$$

Assuming Taylor expansion around ρ_0:

$$S(\langle\rho\rangle) \approx J - L\frac{\rho_0 - \langle\rho\rangle}{3\rho_0} \rightarrow J \approx S(\langle\rho\rangle) + \frac{\rho_0 - \langle\rho\rangle}{3\rho_0} L$$

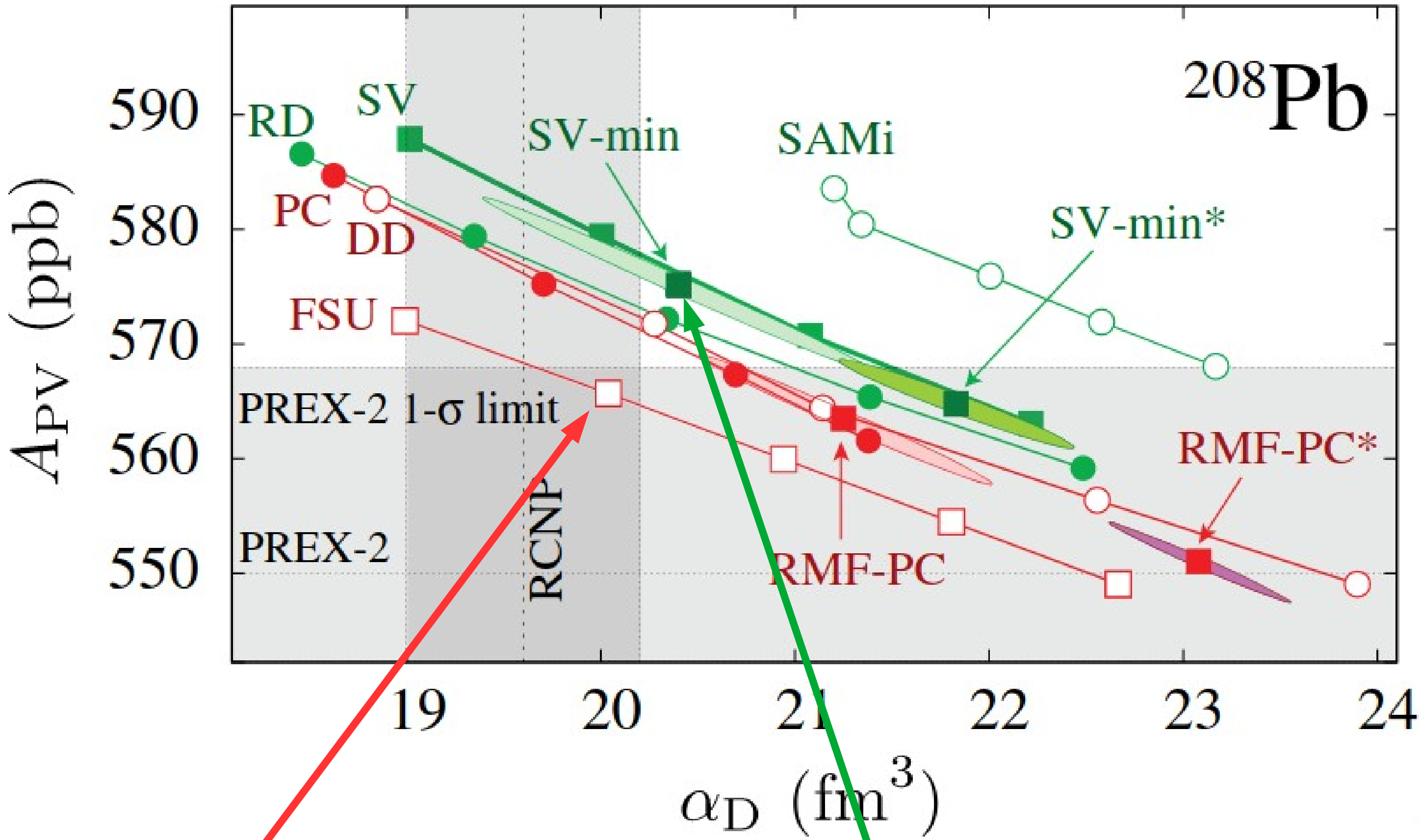
one can qualitatively understand the result!!

$S(\langle\rho\rangle \approx 0.08\ \text{fm}^{-3}) \approx 25$ MeV

X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013

X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

And if we combine A_{PV} and α_D in ^{208}Pb?



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. 127, 232501 (2021)

- Models with an * contain Apv in ^{208}Pb in the fitting protocol

- Models connected by lines: family

- SV, PC and RD models fitted to the same data

SV-min:

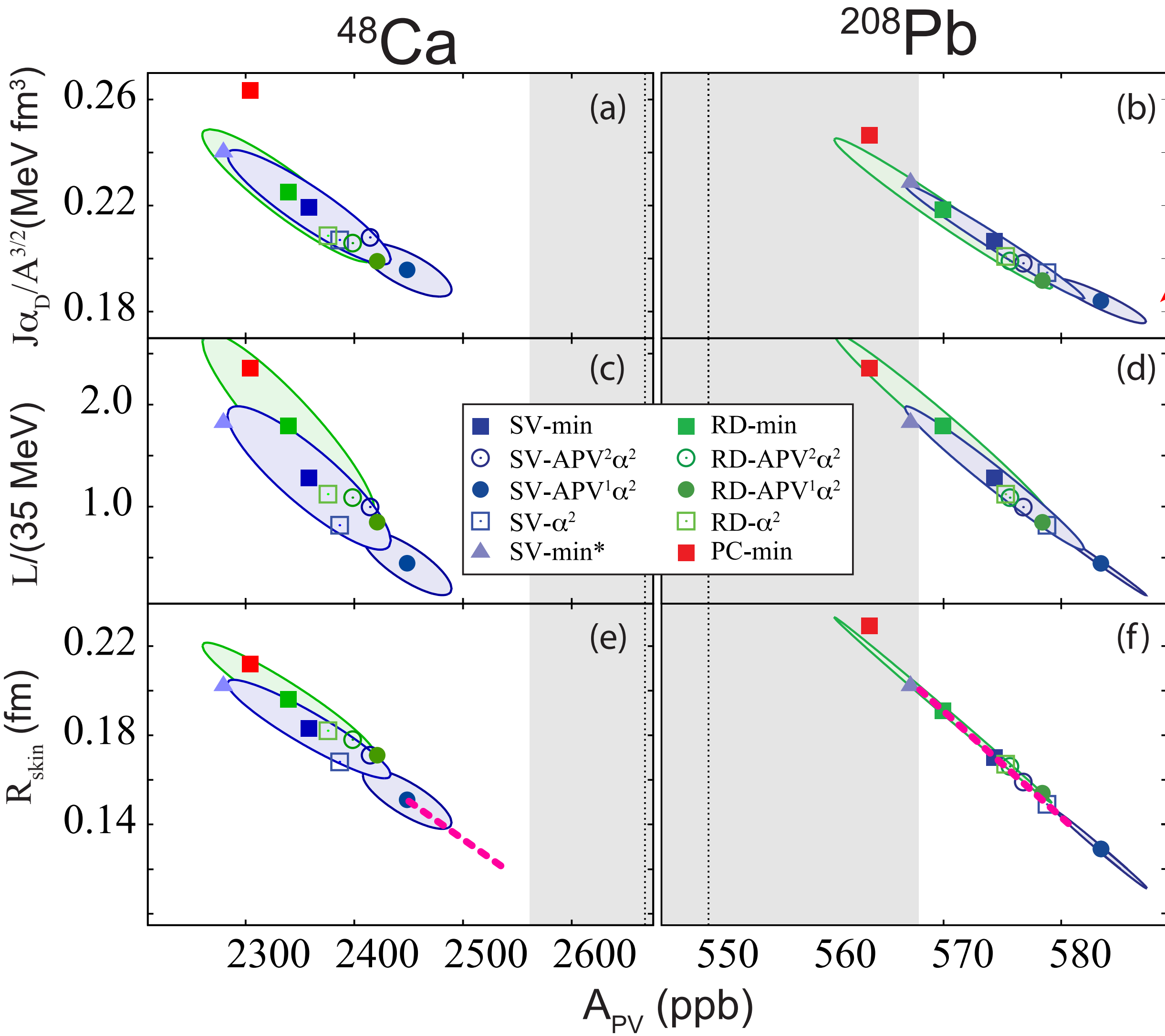
$$r_{\text{skin}} = 0.19 \pm 0.02 \text{ fm}$$

$$L = 54 \pm 8 \text{ MeV}$$

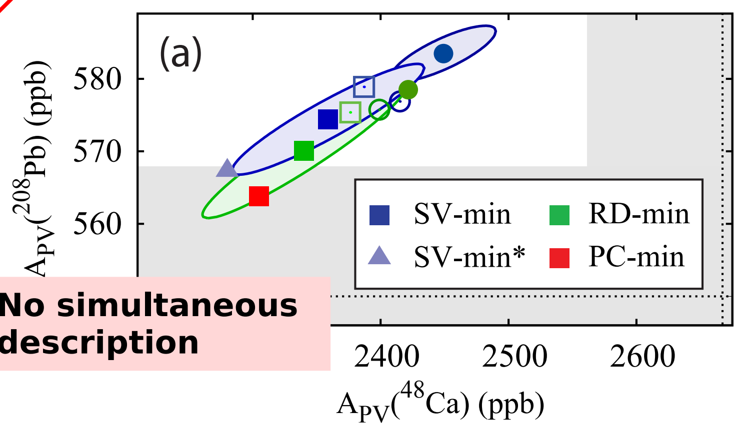
B and **Rch** away from "expected" EDF accuracy for B and Rch

Theoretical and experimental 1σ errors overlap for SV-min
But not overlap both exp.

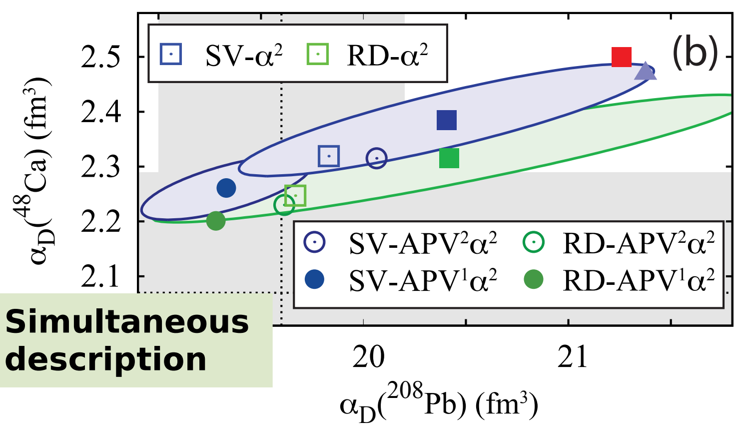
… and in ^{48}Ca and ^{208}Pb



Theoretical (**EDFs** and **ab initio**) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca



No simultaneous description

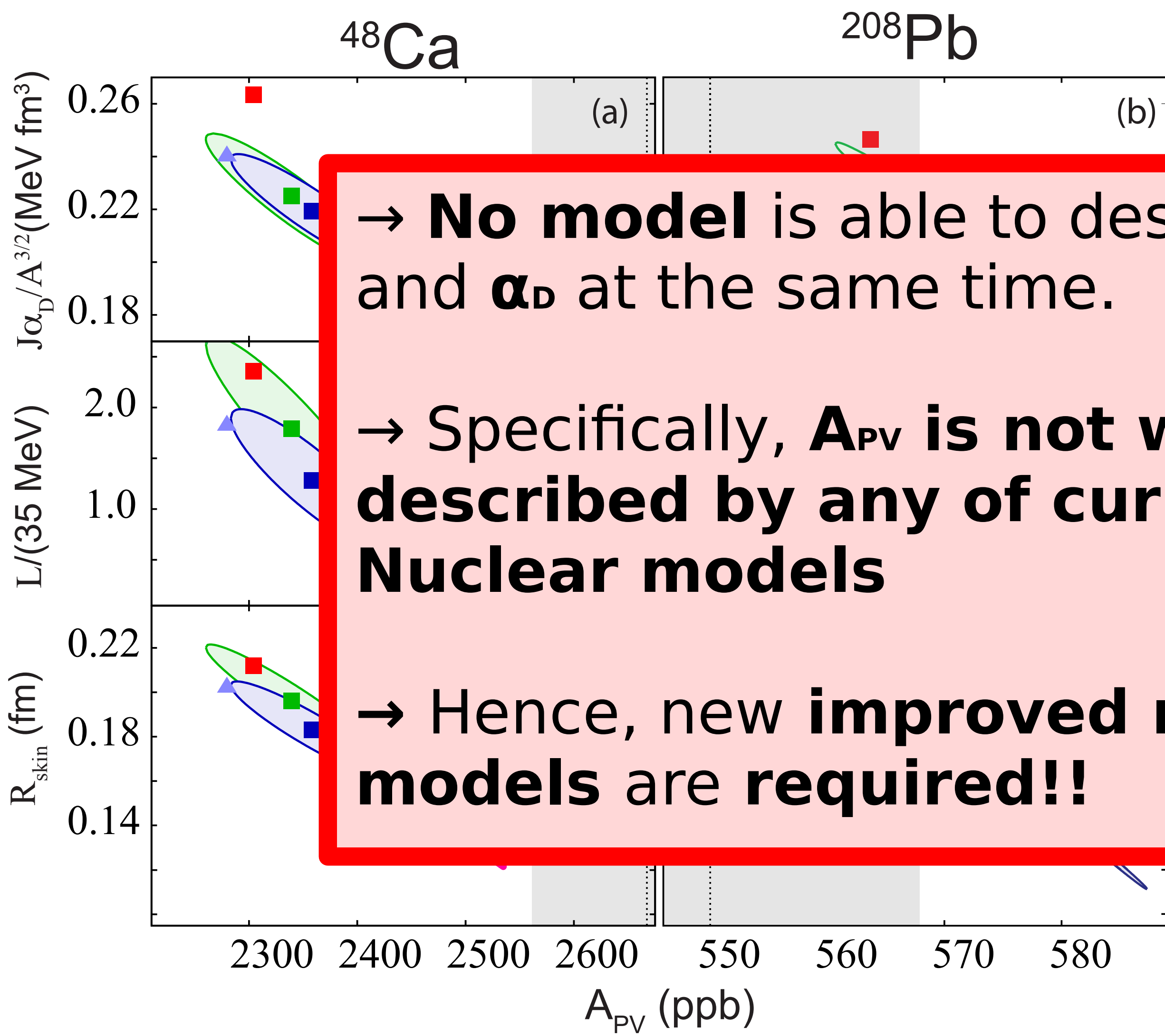


Simultaneous description

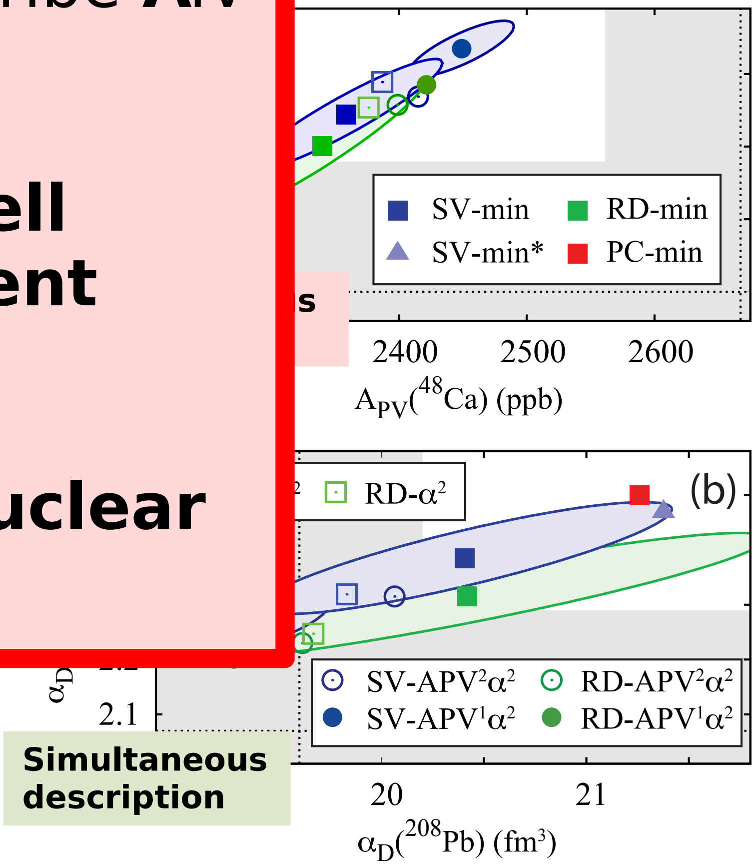
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. 129, 232501 (2022)

Magenta dashed lines from Δr_{np} in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

… and in ^{48}Ca and ^{208}Pb



Theoretical (**EDFs** and **ab initio**) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca



→ **No model** is able to describe **A$_{PV}$** and **α$_D$** at the same time.

→ Specifically, **A$_{PV}$ is not well described by any of current Nuclear models**

→ Hence, new **improved nuclear models** are **required!!**

Simultaneous description

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. 129, 232501 (2022)

Magenta dashed lines from Δr_{np} in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Summary from Progress in Particle and Nuclear Physics 101 (2018) 96–176

EoS par.	Observable	Range	Comments
ρ_0	$\langle r_{ch}^2 \rangle^{1/2}$	0.154–0.159	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
e_0	$M(N,Z)$	-16.2 to -15.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
K_0	$M(N,Z)$	220–245	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	ISGMR	220–260	From EDFs in closed shell nuclei [116]
	ISGMR	250–315	Blaizot's formula [Eq. (32)] [51]
	ISGMR	∼200	EDF describing also open shell nuclei [118]
J	$M(N,Z)$	29-35.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	IVGDR	∼24.1(8) + L/8	From EDF analysis $[S(\rho = 0.1\ \mathrm{fm}^{-3}) = 24.1(8)\ \mathrm{MeV}]$ [273]
	PDS	30.2–33.8	From EDF analysis [370]
	PDS	31.0–33.6	From EDF analysis [371]
	α_D	$24.5(8) + 0.168(7)L$	From EDF analysis ^{208}Pb [96]
	α_D	30–35	From EDF analysis [179]
	IAS and Δr_{np}	30.2–33.7	From EDF analysis [325]
	AGDR	31.2–35.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	32–33	From EDF analysis [508]
	compilation	29.0–32.7	[106]
	compilation	30.7–32.5	[107]
	compilation	28.5–34.9	[3]
L	$M(N,Z)$	27–113	Most accurate EDFs on $M(N,Z)$ $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	ρ_n	40–110	proton-^{208}Pb scattering [24]
	ρ_n	0–60	π photoproduction (^{208}Pb) [181]
	ρ_n	30–80	antiprotonic at. (EDF analysis) [102,509]
	ρ_{weak}	>20	Parity violating scattering [27]
	PDS	32–54	From EDF analysis [370]
	PDS	49.1–80.5	From EDF analysis [371]
	α_D	20–66	From EDF analysis [179]
	IVGQR and ISGQR	19–55	From EDF analysis [101]
	IAS and Δr_{np}	35–75	From EDF analysis [325]
	AGDR	75.2–122.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	45.2–54.6	From EDF analysis [508]
	compilation	40.5–61.9	[106]
	compilation	42.4–75.4	[107]
	compilation	30.6–86.8	[3]

Some alternative compilations:

M. Oertel, M. Hempel, T. Klähn, S. Typel, Rev. Modern Phys. 89 (2017) 015007.

M.B. Tsang, et al., Phys. Rev. C 86 (2012) 015803.

Bao-An Li, Xiao Han, Phys. Lett. B 727 (1) (2013) 276–281.

Summary

with qualitative indication of accuracy needed to describe experiment (note that absolute values might be subject to systematics)

→ $\rho_0 \in [0.154, 0.159]$ fm^{-3} → relative accuracy 2%

→ needed to describe experiment (Rch) ≤0.1%

→ $e_0 \in [15.6, 16.2]$ MeV→ relative accuracy 4%

→ needed to describe experiment (B) ≤0.0001%

→ $K_0 \in [220, 260]$ MeV→ relative accuracy 17%

→ needed to describe experiment (E_x^{GMR}) ≤7%

→ $J \in [30, 35]$ MeV → relative accuracy 15%

→ needed to describe experiment (α) ≤15%

→ $L \in [20, 120]$ MeV → relative accuracy 150%

→ needed to describe experiment (α) ≤50%

→ ...

Conclusion:

→ **Dense neutron matter** ($\rho<2\rho_0$) must be described by **nuclear models**

→ Current **nuclear models do not reproduce A_{PV}** and at the same time other relevant observables

→ **Theory must provide new improved models** for reliable applications in **nuclear structure** and **nuclear reaction** studies as well as in **nuclear astrophysics.**

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