Constraining the nuclear equation of state through nuclear experiments

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Nuclear Equation of State (EoS)

Unpolarized, uniform nuclear matter at T=0 assuming isospin symmetry

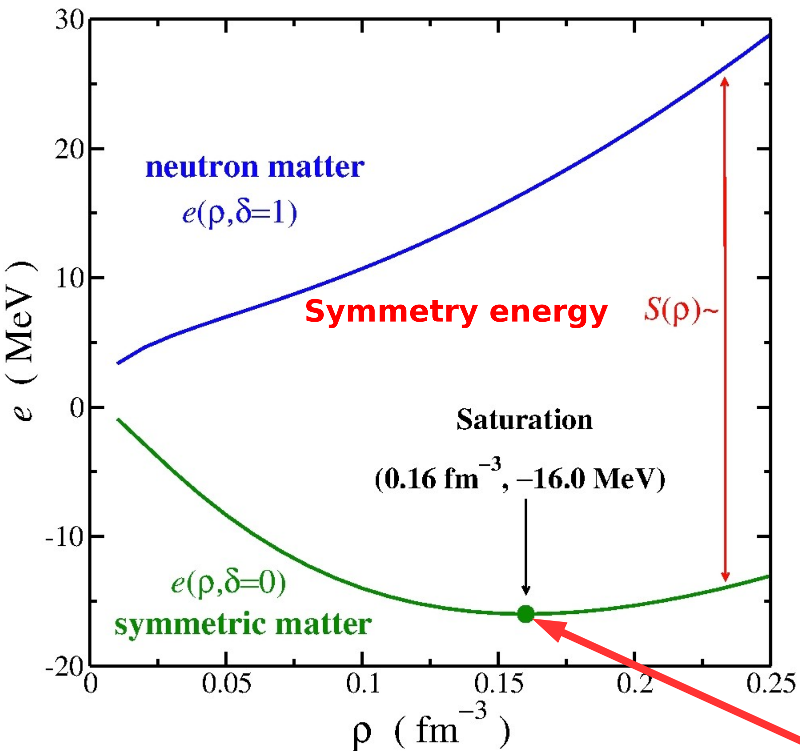
Energy per nucleon (e) as a **function** of the **total density** [$\rho=\rho_n+\rho_p$] and their **relative difference** [$\delta=(\rho_n+\rho_p)/\rho$].

→ Due to isospin symmetry only even powers of δ will appear

→ Stable nuclei tend to show small values of δ

Taylor expansion for δ→ 0:

$$\boxed{e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4]}$$



It is customary to also **expand** e(ρ,0) and S(ρ) around nuclear **saturation density** $\rho_0 \sim 0.16\ \mathrm{fm}^{-3}$

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\mathrm{sym}}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

K_0 → how **compressible** is symmetric matter at ρ_0

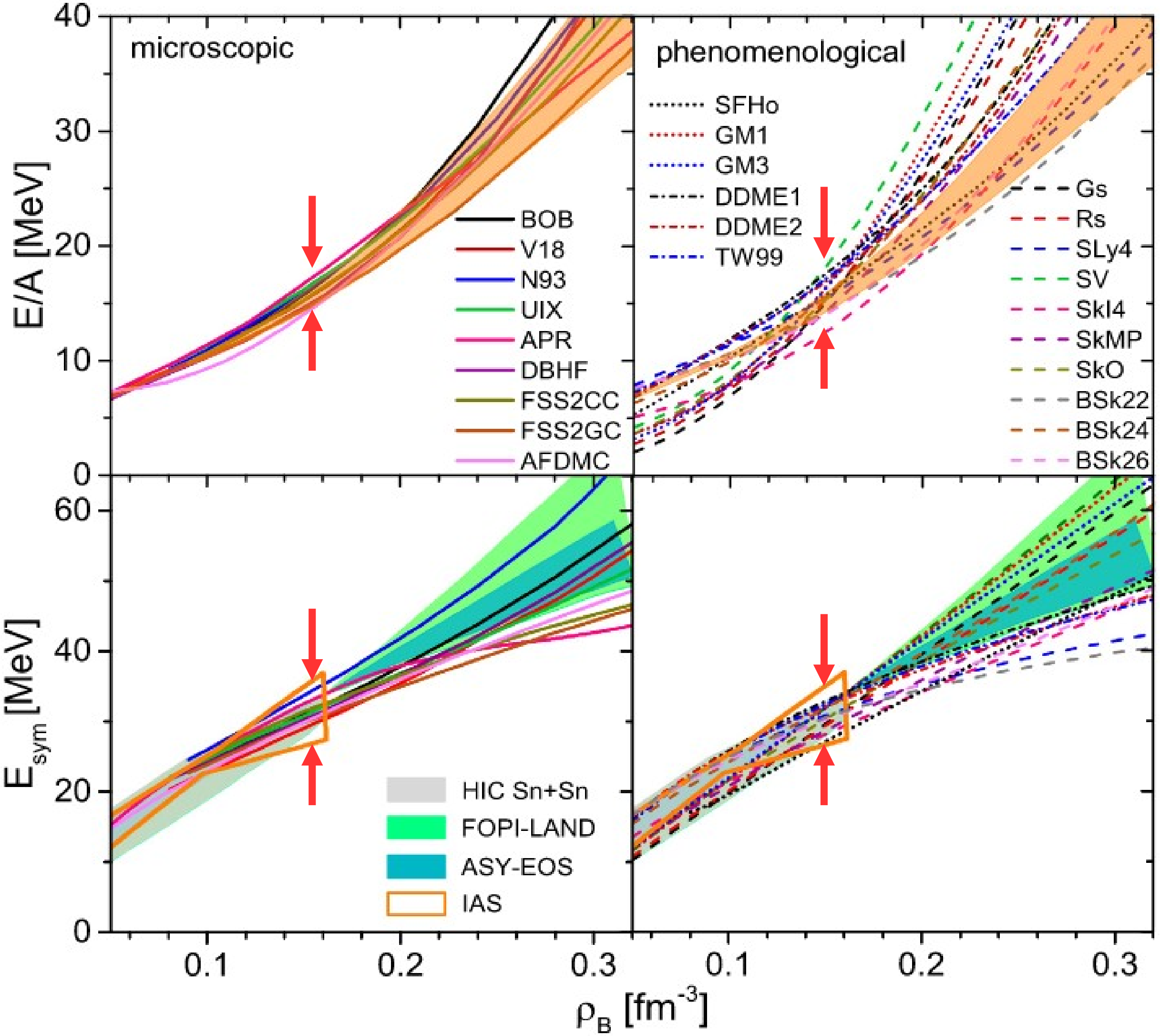
J (a_A) → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L (a_S) → **neutron pressure** in neutron matter at ρ_0

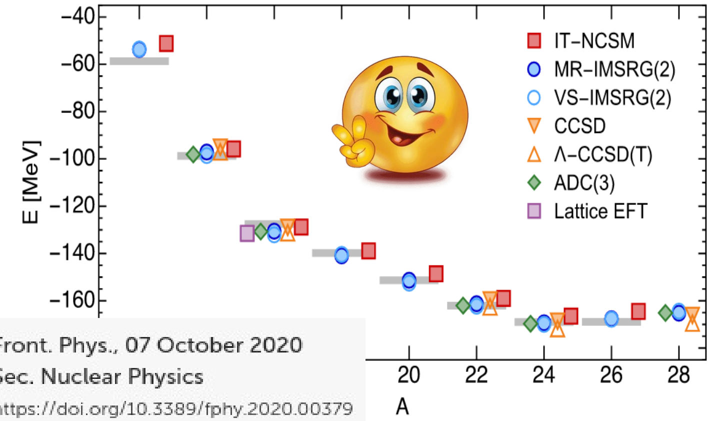
$$P(\rho=\rho_0,\delta=0) = 0\ \ \mathrm{MeV\ fm}^{-3}$$

EoS from current nuclear models

Micorscopic and phenomenological models constrainted by different data display similar discrepances on the EoS

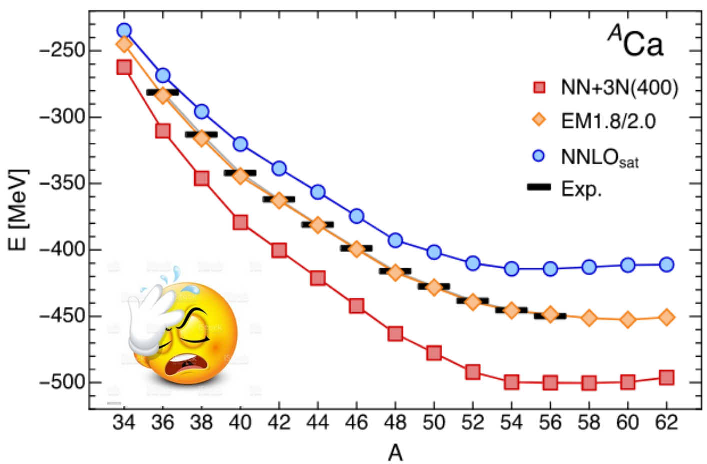


Many-body methods have been shown to agree



Front. Phys., 07 October 2020
Sec. Nuclear Physics
https://doi.org/10.3389/fphy.2020.00379

Main source of uncertainty in the nuclear Hamiltonian



Progress in Particle and Nuclear Physics 120 (2021) 103879

Light orange bands from χEFT from Drischler C., et al. Phys. Rev. Lett., 125 202702 (2020).

Exprimental binding energies and charge radii
are telling us the e_0 and ρ_0 with exquisite accuracy

→ A **small change** in the **saturation density** will **impact** the **size** of the **nucleus. Charge radii** are determined to an average accuracy of 0.016 fm (Angeli 2013).

For example, if one aims at determining the $\mathbf{r_{ch}}$ **= 5.5012±0.0013 fm** in ^{208}**Pb** one must be **very precise** in the determination of $\boldsymbol{\rho_0}$:

$$\boxed{\frac{\delta\rho_0}{\rho_0} = -3\frac{\delta R}{R} \;\rightarrow\; \frac{\delta\rho_0}{\rho_0} \lesssim 0.1\%}$$

<u>Note:</u> typical average theoretical deviation of accurate EDFs ~ 0.02 fm → $\boldsymbol{\delta\rho_0/\rho_0}$ is determined up to about a **1% accuracy** (That is, third digit in ρ_0).

→ In a similar way, a **small change** in the **saturation energy** (about $e_0 \approx$ -16 MeV) will **impact** on the **nuclear masses.**

For example, if one aims at determining the **B = 1636.4296±0.0012 MeV** in ^{208}**Pb** one must be **very precise** in the determination of $\mathbf{e_0}$:

$$\boxed{\frac{\delta B}{B} = \frac{\delta e_0}{e_0} \rightarrow \frac{\delta e_0}{e_0} \lesssim 10^{-6}}$$

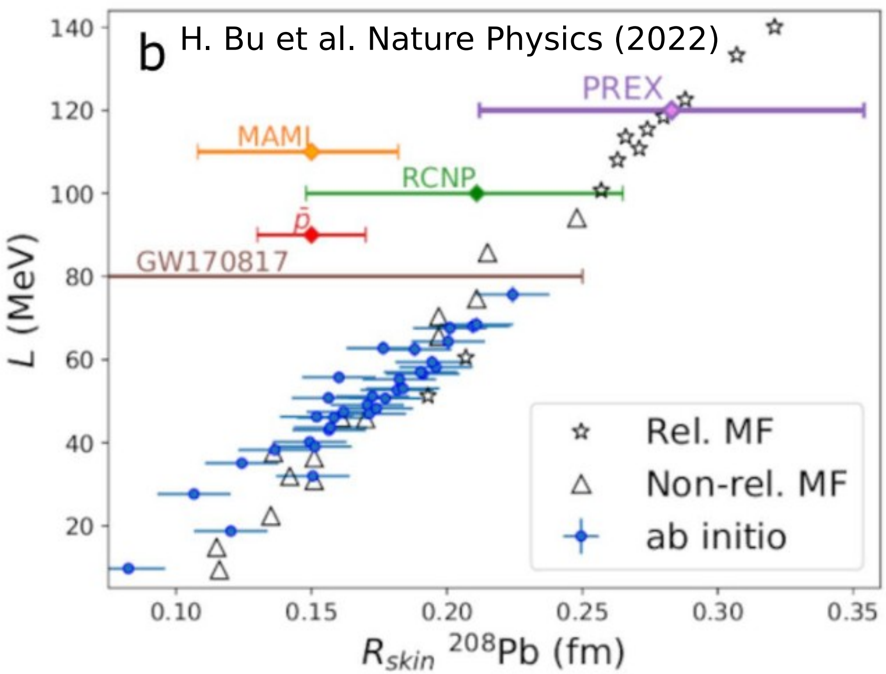
<u>Note:</u> typical average theoretical deviation of accurate EDFs ~ 1-2 MeV → $\mathbf{\delta e_0/e_0}$ is determined up to about a **0.1% accuracy** (That is, second decimal digit in e_0).

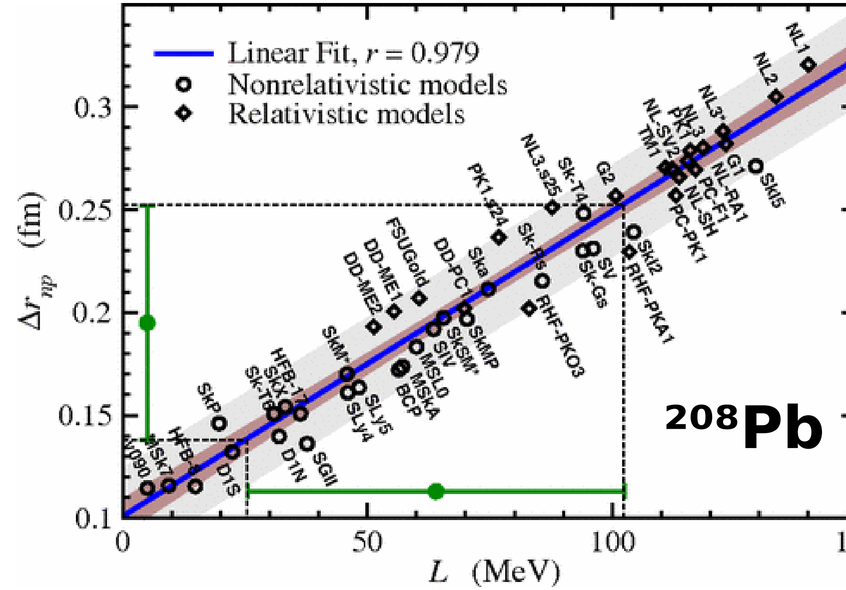
Neutron skin is a good proxy to L

B. Alex Brown Phys. Rev. Lett. 85, 5296 (2000)

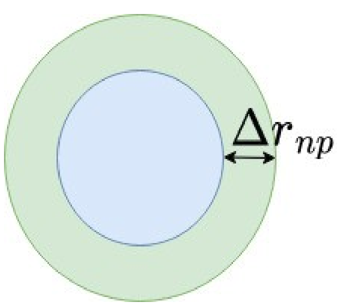
The **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in a heavy neutron rich nucleus is related to the **neutron pressure** ($\delta=1$) around ρ_0 (L).

$$P(\rho_0,\delta) = \rho_0^2 \frac{\partial e(\rho,\delta)}{\partial \rho}\bigg|_{\rho=\rho_0} = \frac{1}{3}\rho_0 \delta^2 L$$





Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



→ From the Droplet Model:

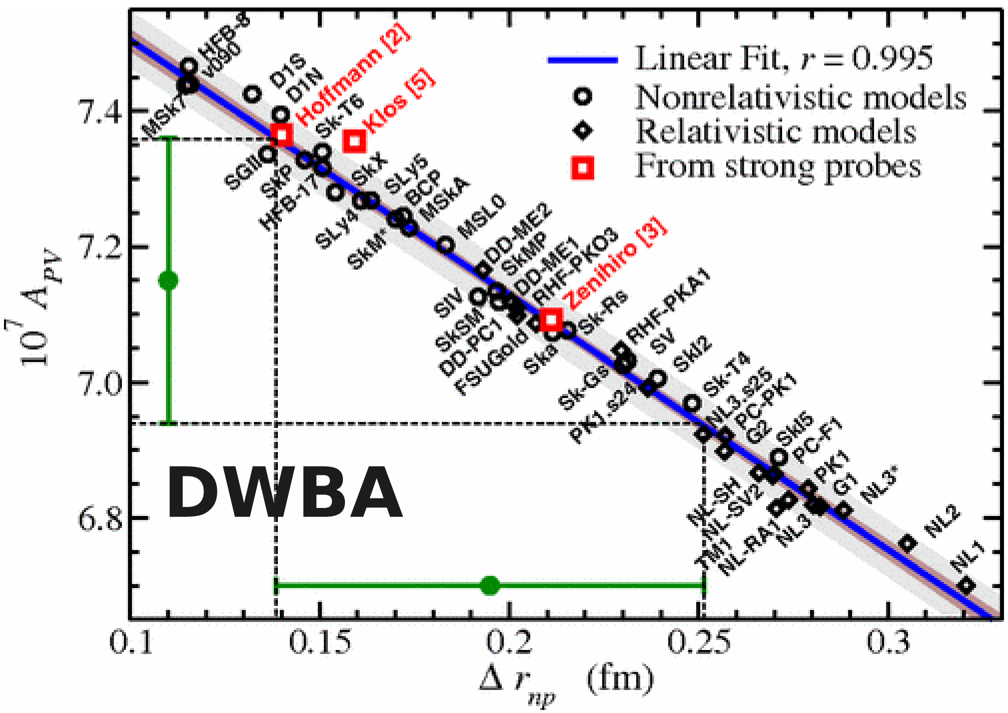
$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

How to measure it? (in a direct way)
Parity violating electron scattering

Polarized electron-Nucleus scattering:

→ In good approximation, **the weak interaction** probes the **neutron distribution** in nuclei while **Coulomb interaction** probes the **proton distribution**

→ **Different experimental efforts @ Jlab (USA) & MAMI (Germany)**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

→ **Electrons** interact by **exchanging** a **γ** (couples to **p**) or a **Z_0** boson (couples to **n**)

→ **Ultra-relativistic electrons**,depending on their helicity (±), will interact with the nucleus seeing a slightly different potential: **Coulomb** ± **Weak**

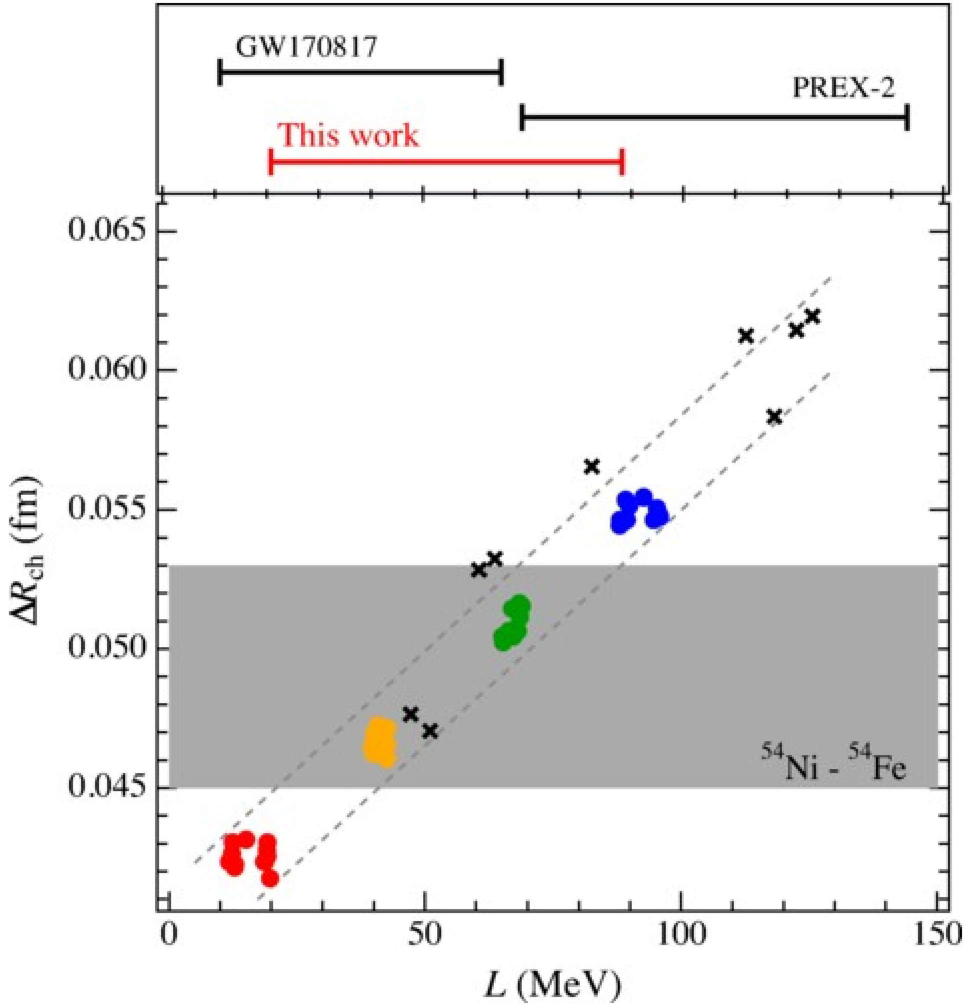
$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

→ Main **unknown** is **ρ_n**

→ In **PWBA** for small momentum transfer **q**:

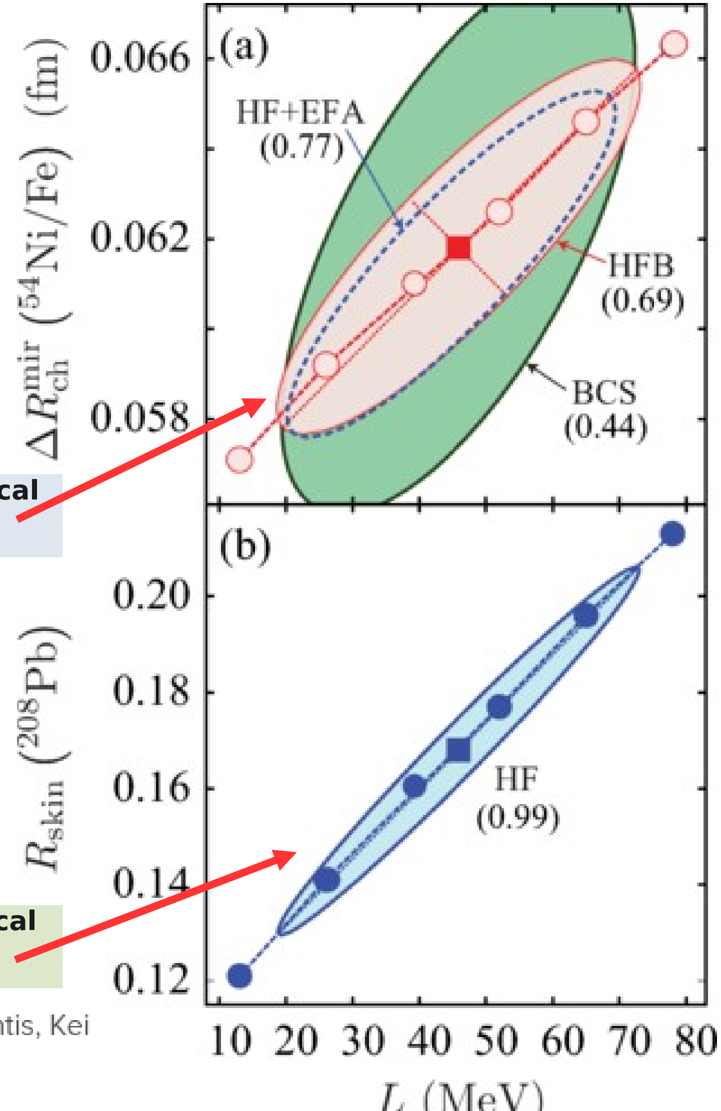
$$A_{pv} = \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 r_p^2}{3F_p(q)}\right)\Delta r_{np}$$

Other proxys to L? (recently proposed)
Δr_{ch} in mirror mass nuclei
Isospin symmetry → $\Delta r_{ch} := r_{ch}(^{54}Ni) - r_{ch}(^{54}Fe) = \Delta r_{np}(^{54}Fe)$



Large theoretical uncertainties

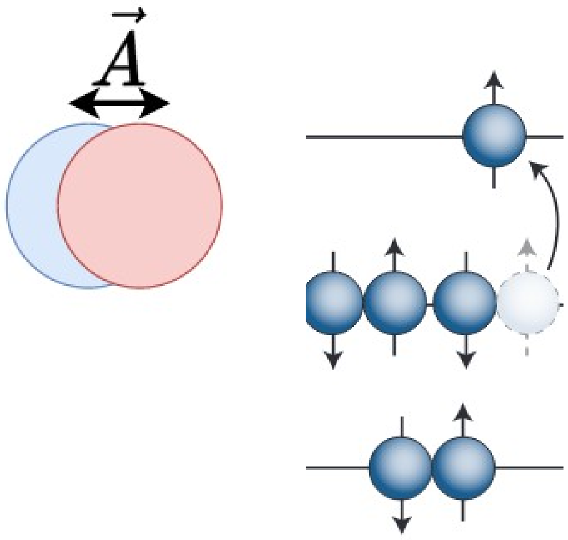
Small theoretical uncertainties



Paul-Gerhard Reinhard and Witold Nazarewicz
Phys. Rev. C **105**, L021301 – Published 3 February 2022

Skyy V. Pineda, Kristian König, Dominic M. Rossi, B. Alex Brown, Anthony Incorvati, Jeremy Lantis, Kei Minamisono, Wilfried Nörtershäuser, Jorge Piekarewicz, Robert Powel, and Felix Sommer
Phys. Rev. Lett. **127**, 182503 – Published 29 October 2021

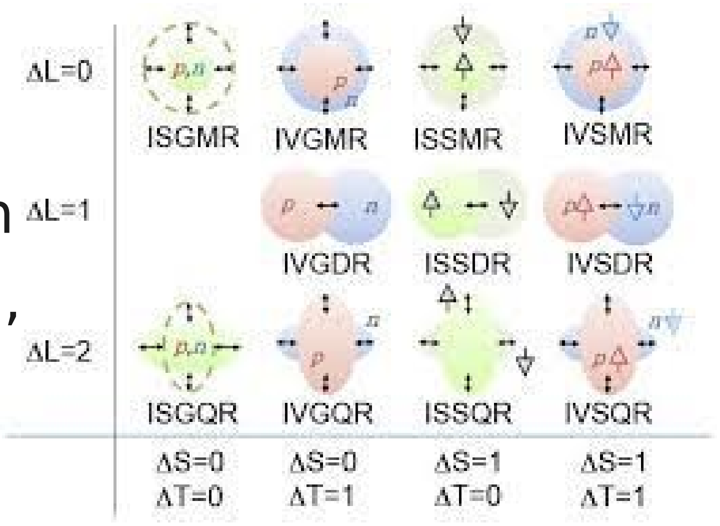
What else?
Giant Resonances



→ **Collective motion** giving rise to **Giant Resonances** could be ***qualitatively*** understood as *macroscopic* **oscillation/vibration** of the **nucleon densities** $\delta\rho \sim \mathbf{A}\cdot\nabla\rho$

→ **Microscopically,** overall Giant Resonance properties have been seen to be well described as a **coherent superposition** of **1 particle – 1 hole** excitations.

→ **Collective excitations** can be **classified** in terms of the quantum numbers **J = L + S** and **parity**, $\pi=(-1)^L$, and modeled by using transition operators of the type $\sum \tau_a\, r^L\, (\mathbf{Y}_{LM} \times \boldsymbol{\sigma})_{JM.}$ with $\tau_a=1$, τ_z for non-charge exchange IS and IV, respectively, or $\tau_a=\tau_\pm$ for charge exchange.



Giant Resonances: some guidance

Non interacting nucleons confined in an harmonic oscillator potential and perturbed with an harmonic perturbation

The **HO Hamiltonian**, in terms of the conjugate variables α (or r) and dα/dt (or v) and the ($C_0 \leftrightarrow m\omega^2$) and ($B_0 \leftrightarrow m$) parameters, could be written as (Bohr&Mottelson):

$$\mathcal{H}_0 = \frac{1}{2}B_0\dot{\alpha}^2 + \frac{1}{2}C_0\alpha^2 \rightarrow E_0 = \hbar\left(\frac{C_0}{B_0}\right)^{1/2}$$

$$E = \left(\frac{1}{B_0}\frac{\partial^2 U}{\partial \alpha^2}\right)^{1/2}$$

Assume harmonic perturbation (restoring force)

$$F = -\kappa\alpha \rightarrow V = \frac{1}{2}\kappa\alpha^2$$

$$\mathcal{H} = \mathcal{H}_0 + V \rightarrow \mathcal{H} = \frac{1}{2}B_0\dot{\alpha}^2 + \frac{1}{2}(C_0+\kappa)\alpha^2$$

$$E = E_0\left(1 + \frac{\kappa}{C_0}\right)^{1/2}$$

Restoring force

Depending on the type of perturbation

$$\boxed{\begin{gathered} E \propto \left(\frac{\partial^2 e}{\partial \delta^2}\right)^{1/2} = [S(\rho)]^{1/2} \\ E \propto \left(\frac{\partial^2 e}{\partial \rho^2}\right)^{1/2} = [K]^{1/2} \end{gathered}}$$

Sum rules:
Ground state gives access to overall excited properties!!

→ **Sum Rules** or moments of the strength function S(E) sometimes **model independent** ([F,V]=0) and/or predict **very simple expressions** that can be exploited to better **understand the nuclear phenomenology.**

k-moment of S(E):

$$m_k \equiv \int dE E^k S(E)$$

$$m_0 = \sum_\nu |\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger F|0\rangle$$

$$m_1 = \sum_\nu (E_\nu - E_0)|\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger[\mathcal{H}, F]|0\rangle$$

$$m_2 = \sum_\nu (E_\nu - E_0)^2|\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger[\mathcal{H}, [\mathcal{H}, F]]|0\rangle$$

$$m_3 = \sum_\nu (E_\nu - E_0)^3|\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger[\mathcal{H}, [\mathcal{H}, [\mathcal{H}, F]]]|0\rangle$$

Thouless Th→
m_1(RPA) =1/2 <HF|D.C.|HF>

Expectation value of commutators in the G.S.!!

Example for non-spin flip resonances:

$$m_1^{IS} = \frac{A}{4\pi}\frac{\hbar^2}{2m}\frac{2J+1}{A}\int d\vec{r}\left[\left(\frac{df}{dr}\right)^2 + J(J+1)\left(\frac{f}{r}\right)^2\right]\rho(r)$$

$$\boxed{f = r^L}$$

$$m_1^{IV} = m_1^{IS}(1+\mathcal{K})$$

κ ↔ [F,V] non zero

Dielectric theorem:

Inverse Energy Weighted Sum Rule m_{-1} (polarizability)

Ground state |0> **perturbed** by an **external field** λF (λ → 0) so that perturbation theory holds → The **expectation value** of the **Hamiltonian** <H> and of the **operator** <F> can be written:

$$\delta\langle\mathcal{H}\rangle = \lambda^2 \sum_{\nu\neq 0} \frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^3) = \lambda^2 m_{-1} + \mathcal{O}(\lambda^3)$$

$$\delta\langle F\rangle = -2\lambda \sum_{\nu\neq 0} \frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^2) = -2\lambda m_{-1} + \mathcal{O}(\lambda^2)$$

$$\boxed{m_{-1} = \frac{1}{2}\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\lambda^2}\Big|_{\lambda=0} = -\frac{1}{2}\frac{\partial\langle F\rangle}{\partial\lambda}\Big|_{\lambda=0}} \longrightarrow \frac{1}{m_{-1}} = 2\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\langle F\rangle^2}$$

Giant Monopole Resonance and K

Constrained energy ($E^2 := m_1/m_{-1}$) as a proxy of the resonance excitation energy.

The **operator** leading to **monopole transitions** (isotropic changes in the volume if we think about a liquid drop) cannot depend on the orbital angular momentum or spin:

$$F = \sum_{i=1}^{A} r_i^2 \quad (*)$$

Isotropic harmonic perturbation!

The **m_1** and **m_{-1} moments** are:

$$m_1 = \frac{2\hbar^2}{m}\langle r^2\rangle \qquad \frac{1}{m_{-1}} = 2\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\langle r^2\rangle^2}$$

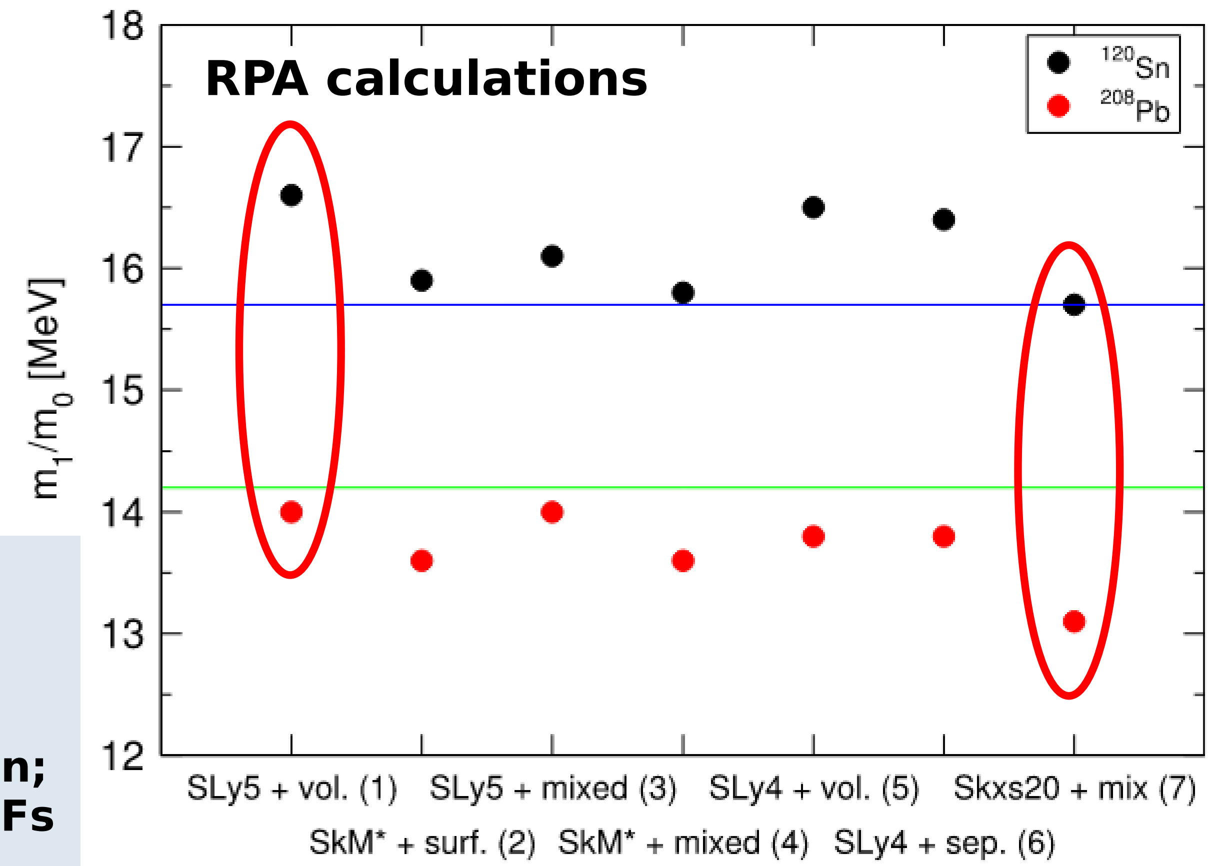
Therefore,

$$\boxed{\left(E_x^{\mathrm{ISGMR}}\right)^2} = \frac{m_1}{m_{-1}} = 4\frac{\hbar^2}{m}\langle r^2\rangle\frac{\partial^2 E}{\partial\langle r^2\rangle^2} = 4A\frac{\hbar^2}{m\langle r^2\rangle}\langle r^2\rangle^2\frac{\partial^2(E/A)}{\partial\langle r^2\rangle^2} \equiv \boxed{K_A\frac{\hbar^2}{m\langle r^2\rangle}}$$

The **excitation energy of the ISGMR** must grow this the square root of the incompressibility, exactly as in the HO model.

(*) other definitions could differ by a factor 4π depending if $Y_{00} = 1/\mathrm{sqrt}(4\pi)$ included or not

Giant Monopole Resonance
do we understand it?



Exp.

$K_0 < \mathbf{240}$ MeV

(Open shell)

Exp.

$K_0 = \mathbf{240 \pm 20}$ MeV

(Closed shell)

Further studies are needed (uncertainty quantification; Extended EDFs & ab initio)

Pairing effects studied here

U. Garg, G. Colò / Progress in Particle and Nuclear Physics 101 (2018) 55–95

Dipole polarizability, J and Δr_{np}

The dipole **polarizability** measures the **tendency** of the nuclear **charge** distribution to be **distorted.**

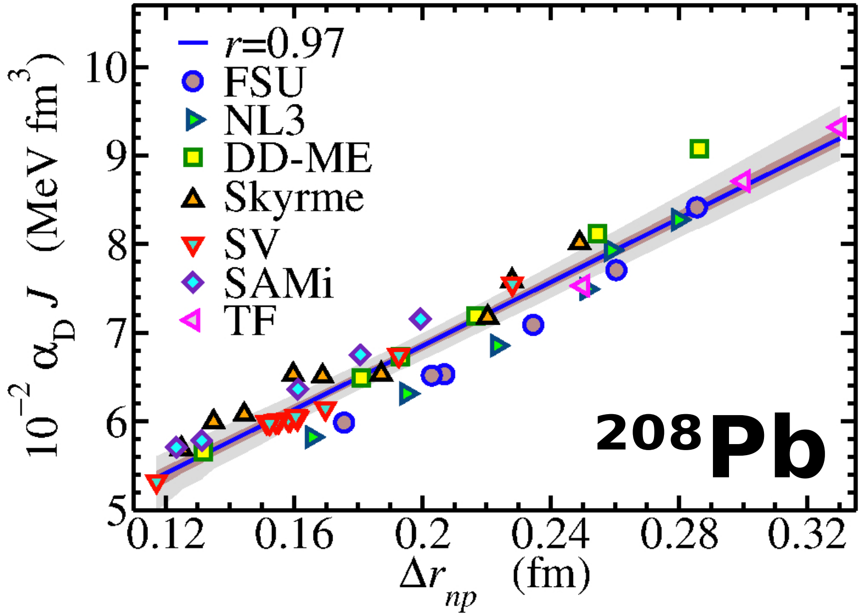
From a macroscopic point of view **α ~ (electric dipole moment)/($E_{external}$)**

→ For guidance, using the **dielectric theorem**, the polarizability can be calculated assuming the **Droplet model**:

$$\alpha_D = \frac{8\pi e^2}{9} m_{-1}(E1)$$

Meyer et al. NPA385 (1982) 269-284

$$\alpha_D \approx \frac{\pi e^2}{54} \frac{\langle r^2 \rangle}{J} A \left(1 + \frac{5}{2} \frac{\Delta r_{np} - \Delta r_{np}^{\text{surf}} - \Delta r_{np}^{\text{Coul}}}{\langle r^2 \rangle^{1/2} (I - I_{\text{Coul}})} \right)$$

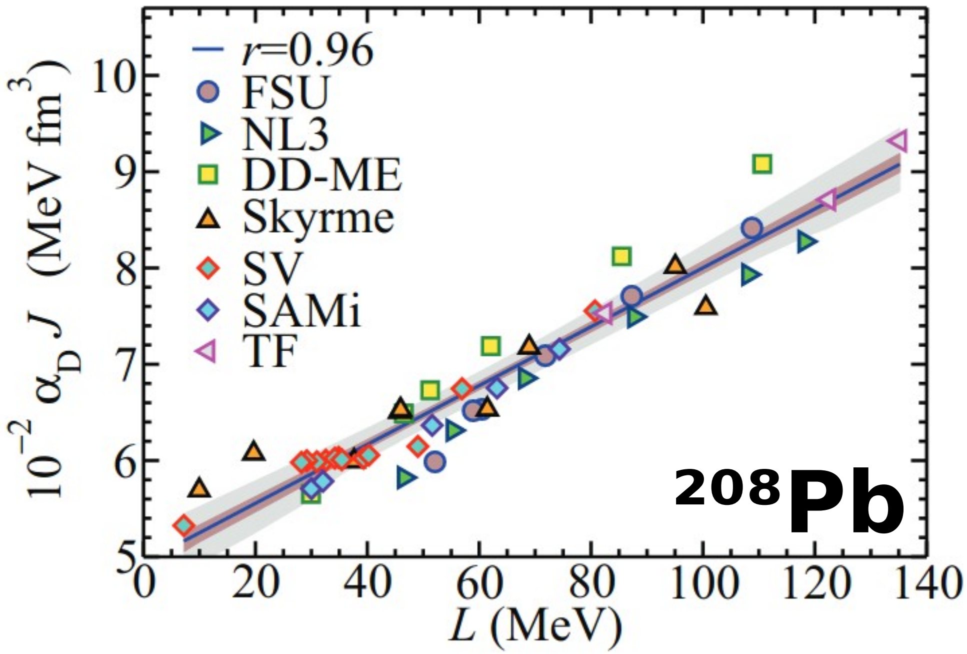


Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C 88, 024316 (2013)

Polarizability increases with the mass (for the dipole $A^{5/3}$, for the quadrupole $A^{7/3}$ and so on …) and it **sets a relation between the EoS parameters J and L**

Dipole polarizability, J and L

Determination of the J vs L relation from experimental data according to EDFs



$$J = 25.0(2) + 0.19(2)L \quad \text{for } ^{68}\text{Ni},$$
$$J = 25.4(1.1) + 0.17(1)L \quad \text{for } ^{120}\text{Sn},$$
$$J = 24.5(8) + 0.17(1)L \quad \text{for } ^{208}\text{Pb}.$$

Assuming Taylor expansion around ρ_0:

$$S(\langle\rho\rangle) \approx J - L\frac{\rho_0 - \langle\rho\rangle}{3\rho_0} \rightarrow J \approx S(\langle\rho\rangle) + \frac{\rho_0 - \langle\rho\rangle}{3\rho_0}L$$

one can qualitatively understand the result!!

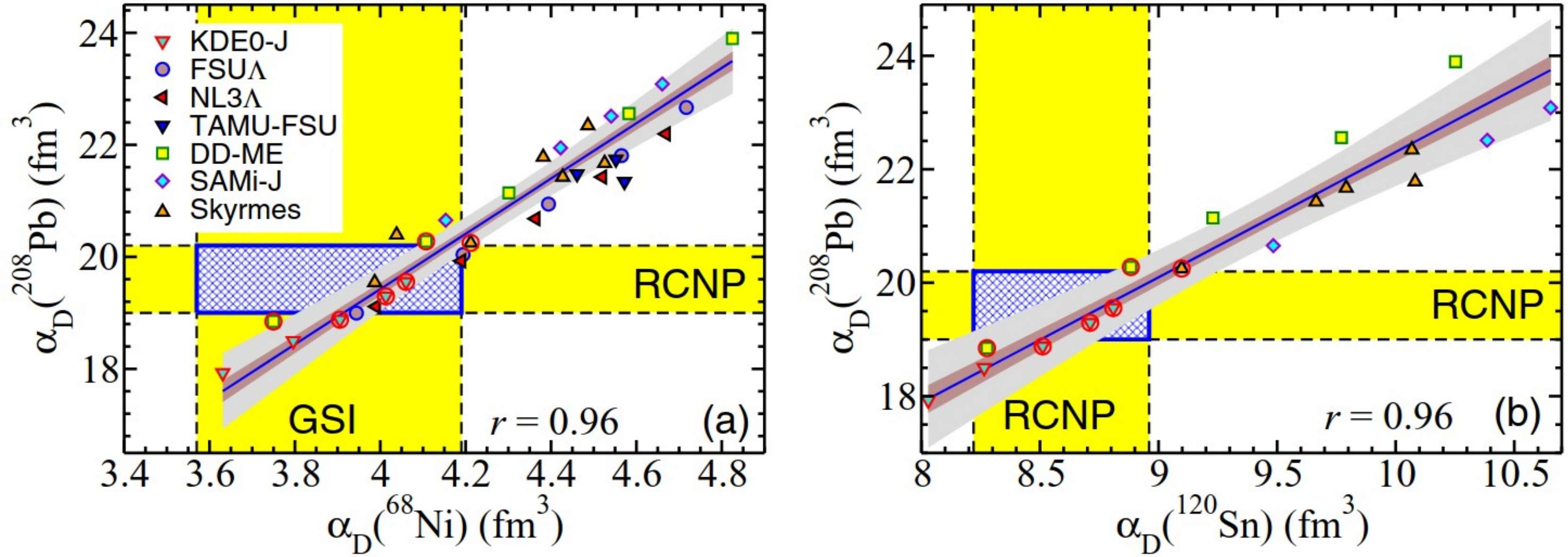
$$S(\langle\rho\rangle \approx 0.08\ \text{fm}^{-3}) \approx 25\ \text{MeV}$$

X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013

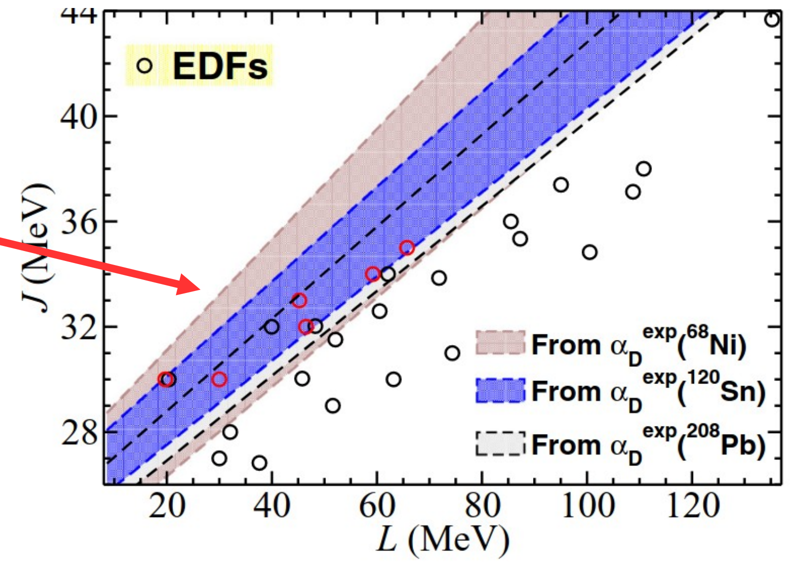
X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

Dipole polarizability, J and L

Selection of EDFs (red circles) compatible with experimental data



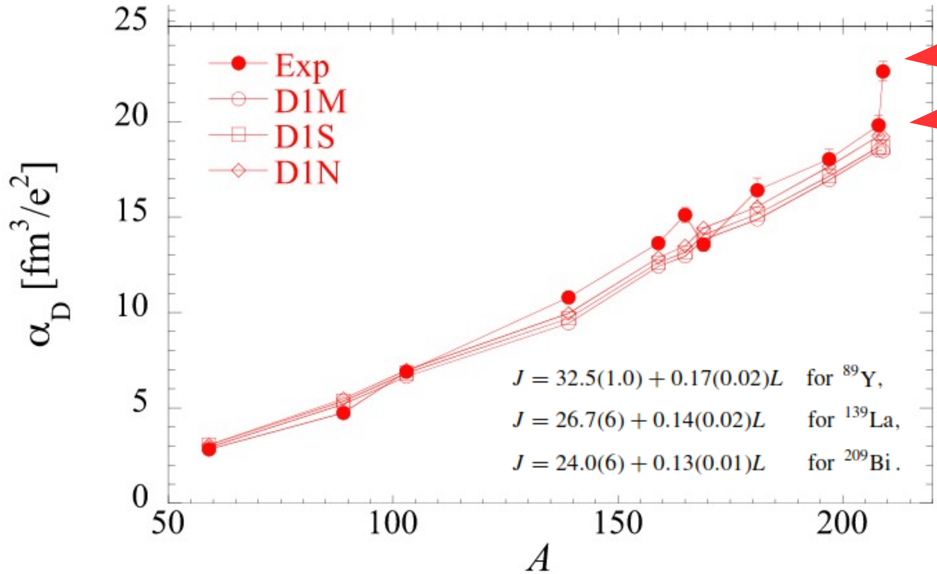
Warning: Some EDFs not compatible with experiment (black circles) are within bands, but most of them outside!



X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013
X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

Dipole polarizability:
do we understand it?

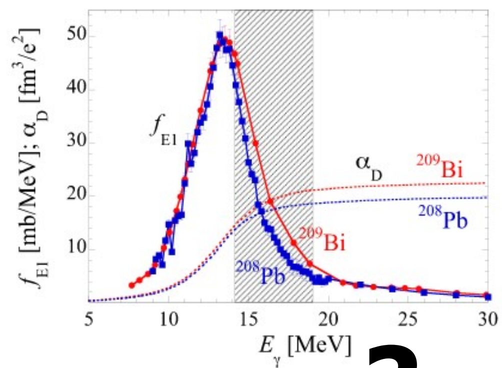
S. Goriely, S. Péru, G. Colò, X. Roca-Maza, I. Gheorghe, D. Filipescu, and H. Utsunomiya
Phys. Rev. C **102**, 064309 – Published 9 December 2020



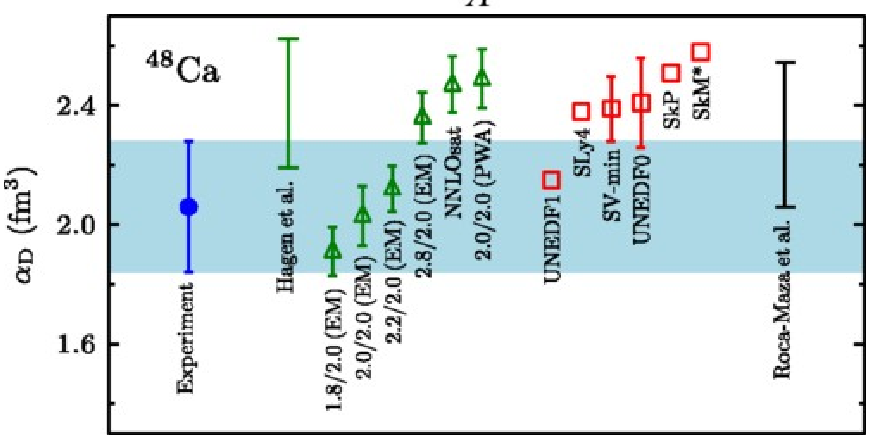
^{209}**Bi**

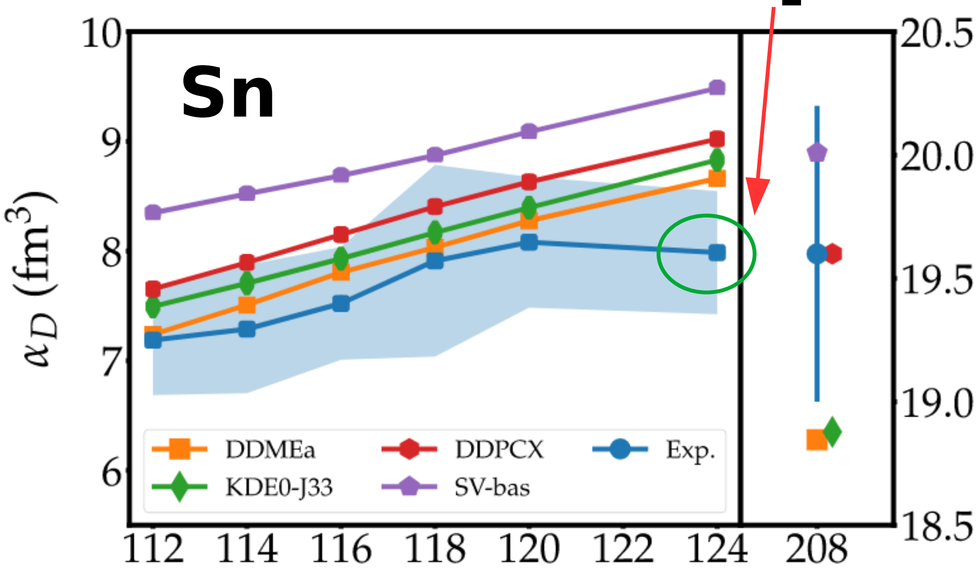
^{208}**Pb**

?



?



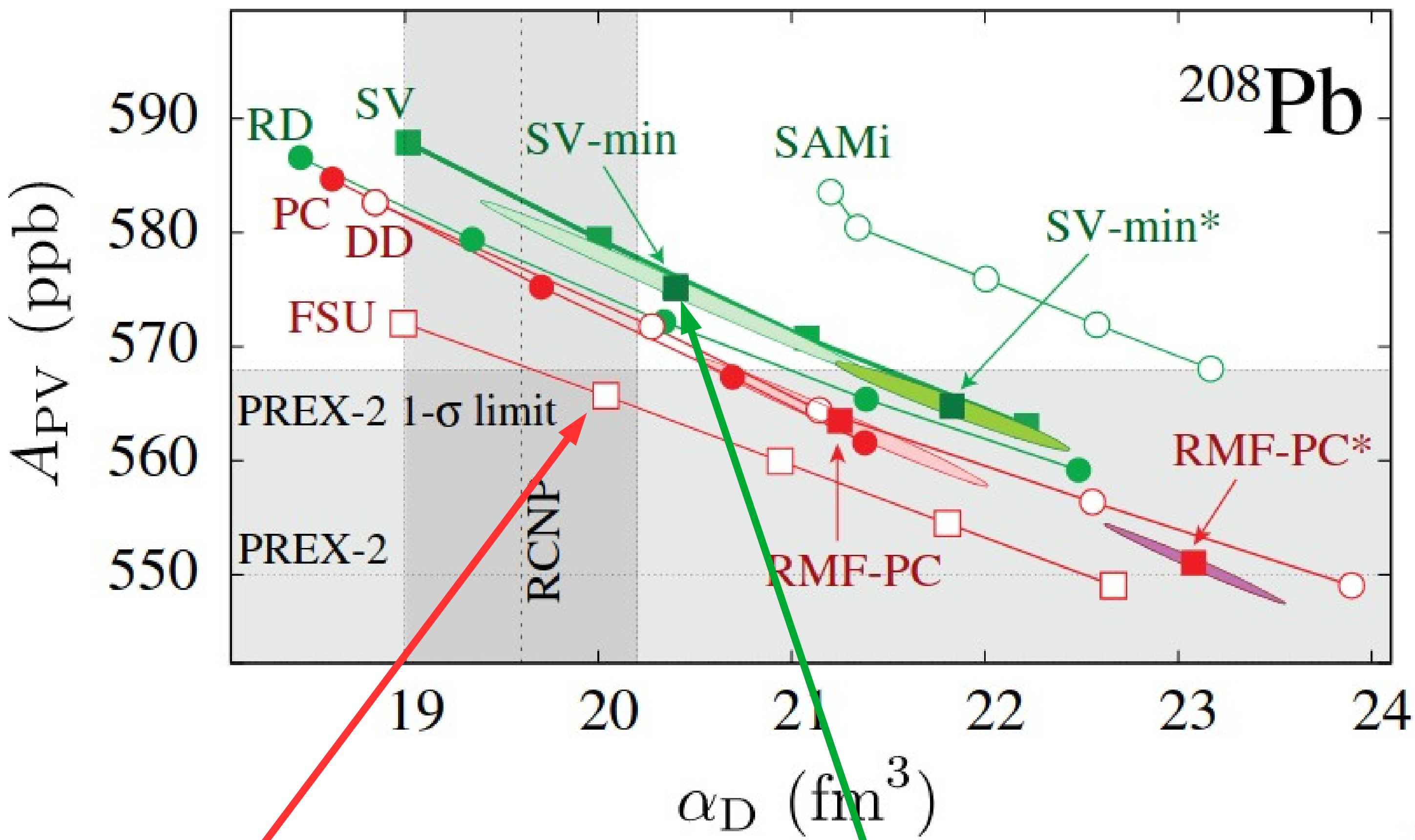


S. Bassauer, P. von Neumann-Cosel, P.-G. Reinhard et al.

J. Birkhan, M. Miorelli, S. Bacca, S. Bassauer, C. A. Bertulani, G. Hagen, H. Matsubara, P. von Neumann-Cosel, T. Papenbrock, N. Pietralla, V. Yu. Ponomarev, A. Richter, A. Schwenk, and A. Tamii
Phys. Rev. Lett. **118**, 252501 – Published 23 June 2017

Physics Letters B 810 (2020) 135804

And if we combine A_{PV} and α_D in ^{208}Pb?



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. 127, 232501 (2021)

- Models with an * contain Apv in ^{208}Pb in the fitting protocol

- Models connected by lines: family

- SV, PC and RD models fitted to the same data

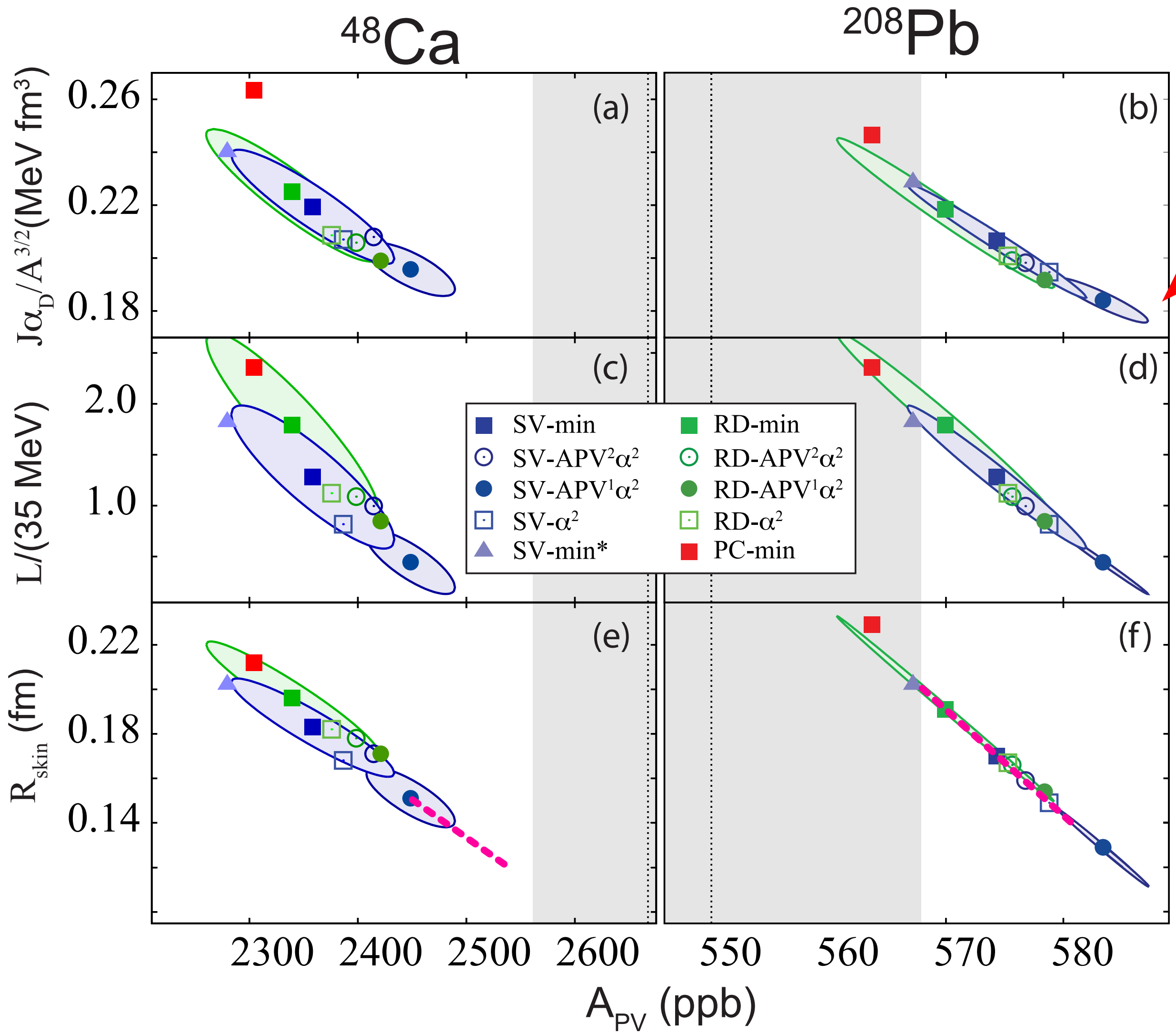
SV-min:

$$r_{\rm skin} = 0.19 \pm 0.02 \text{ fm}$$
$$L = 54 \pm 8 \text{ MeV}$$

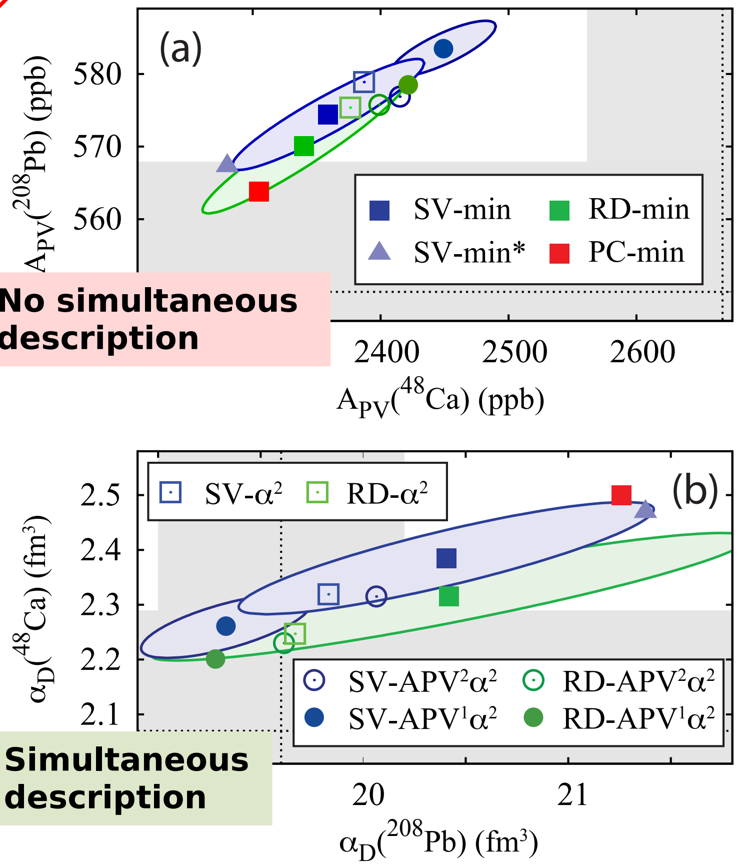
B and **Rch** away from “expected” EDF accuracy for B and Rch

Theoretical and experimental 1σ errors overlap for SV-min
But not overlap both exp.

… and in ^{48}Ca and ^{208}Pb



Theoretical (**EDFs** and **ab initio**) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca



No simultaneous description

Simultaneous description

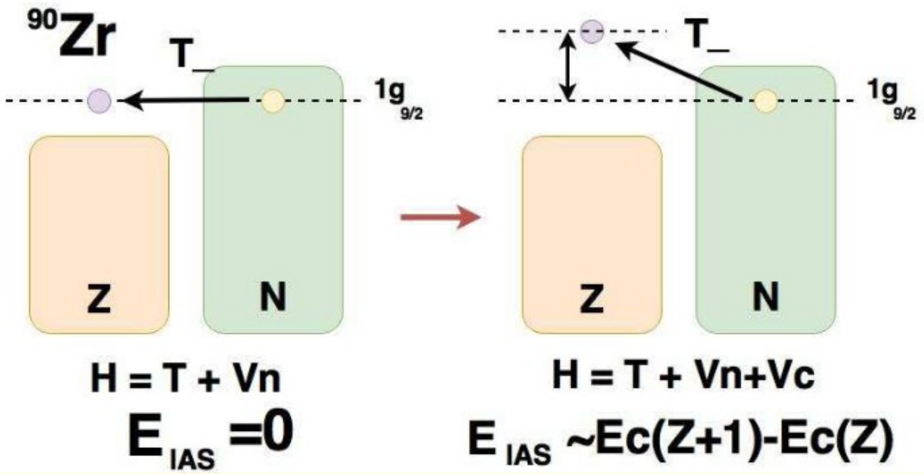
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz arXiv:2206.03134 (2022)

Magenta dashed lines from Δr_{np} in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Isobaric Analog State

Promising observable?

$$F = T_\pm = \sum_i^A t_\pm(i)$$



→ non-energy weighted sum rule:

$$m_0^- - m_0^+ = \langle 0|T_+T_-|0\rangle - \langle 0|T_-T_+|0\rangle$$
$$= \langle 0|[T_+, T_-]|0\rangle = \langle 0|2T_z|0\rangle$$
$$= N - Z$$

→ energy weighted sum rule:

$$m_1 = \sum_\nu (E_\nu - E_0)|\langle \nu|F|0\rangle|^2 = \langle 0|T_+[\mathcal{H}, T_-]|0\rangle$$

[H,T₋] different from zero only if **H** contains terms that **breaks isospin invariance**

→ Hence, the **centroid energy** m_1/m_0 (neglecting isospin mixing, i.e., $\langle 0|T_-T_+|0\rangle \approx 0$):

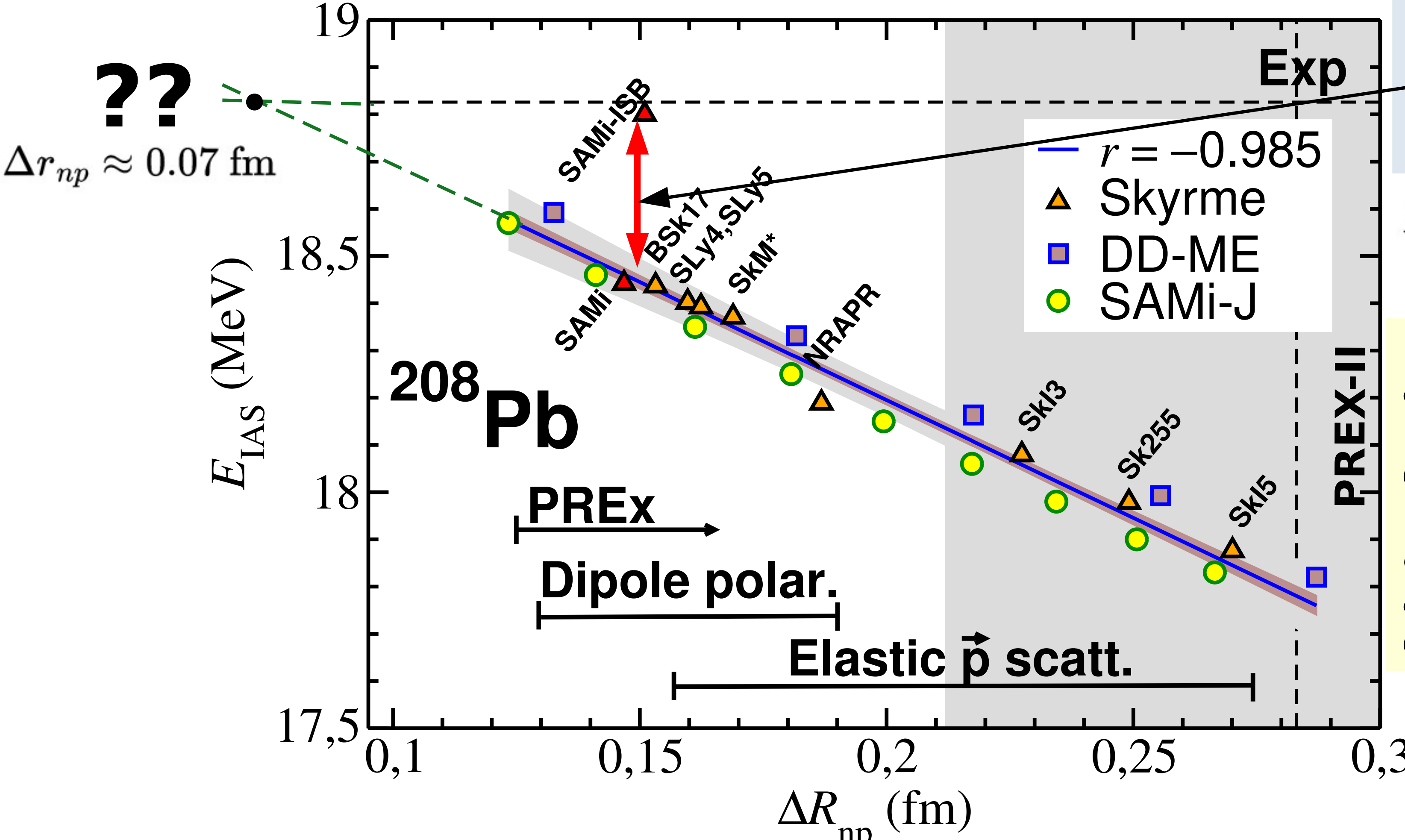
$$E_{\rm IAS} = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle} = \frac{1}{N-Z}\langle 0|T_+[\mathcal{H}, T_-]|0\rangle$$

→ Assuming a simple model: **indepenent particle** model with only **Coulomb breaking isospin symmetry** (neglect exchange effects)

$$E_{\rm IAS}^{\rm C,direct} = \frac{1}{N-Z}\int \left[\rho_n(\vec{r}) - \rho_p(\vec{r})\right] U_{\rm C}^{\rm direct}(\vec{r})d\vec{r}$$

$$E_{\rm IAS} \approx E_{\rm IAS}^{\rm C,direct}$$
$$\approx \frac{6}{5}\frac{Ze^2}{R_p}\left(1 - \frac{1}{2}\frac{N}{N-Z}\frac{R_n - R_p}{R_p}\right)$$
$$\approx \frac{6}{5}\frac{Ze^2}{r_0 A^{1/3}}\left(1 - \sqrt{\frac{5}{12}}\frac{N}{N-Z}\frac{\Delta R_{\rm np}}{r_0 A^{1/3}}\right)$$

Isobaric Analog State, ISB and Δr_{np}



Isospin symmetry breaking in the nuclear force

$E_{\text{IAS}} \sim \langle T_+[\mathcal{H}, T_-] \rangle$

Further studies are needed (uncertainty quantification; Extended EDFs & ab initio guidance)

X. Roca-Maza, G. Colò, and H. Sagawa
Phys. Rev. Lett. **120**, 202501 – Published 18 May 2018

Spin Dipole Resonance and Δr_{np}

$$\sum_{i=1}^{A} \sum_M t_\pm(i) r_i^L \left[Y_L(\hat{r}_i) \otimes \boldsymbol{\sigma}(i)\right]_{JM}$$

→ Non-energy weighted sum-rule (m_0):

$$m_0(t_-) - m_0(t_+) = \langle 0|O_-^{\mathrm{SD}}|0\rangle - \langle 0|O_+^{\mathrm{SD}}|0\rangle$$

$$= \langle 0|[O_-^{\mathrm{SD}}, O_+^{\mathrm{SD}}]|0\rangle = \frac{9}{4\pi}\sum_{i=1}^{A}\langle 0|r_i^2[t_-(i), t_+(i)]|0\rangle$$

$$= 2\frac{9}{4\pi}\sum_{i=1}^{A}\langle 0|r_i^2 t_z(i)|0\rangle = \boxed{\frac{9}{4\pi}\left(N\langle r_n^2\rangle - Z\langle r_p^2\rangle\right)}$$

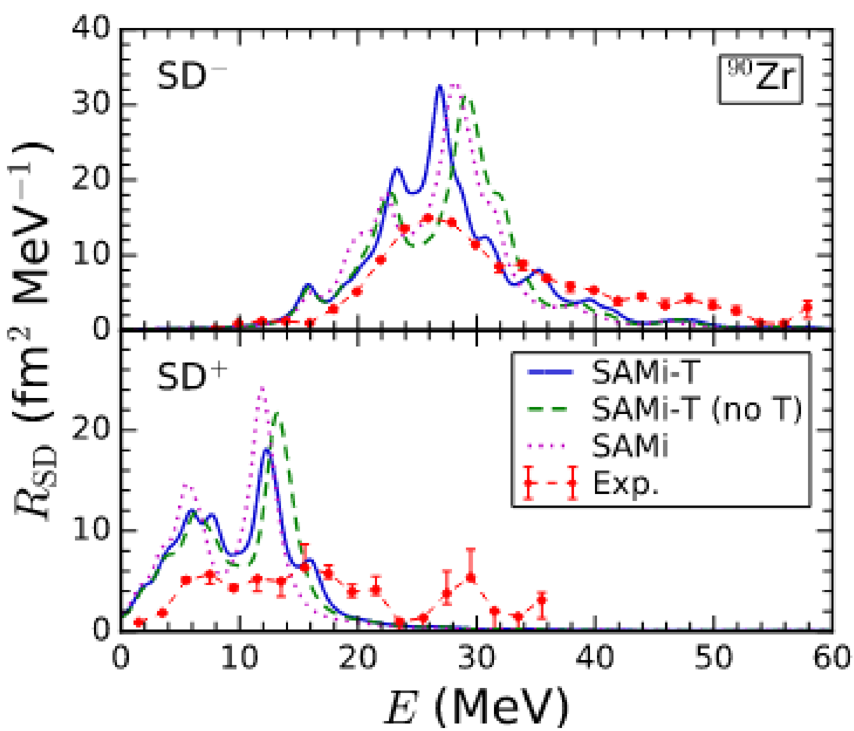
→ Rewritting it in terms of the neutron skin thickness: $\Delta r_{np} = \langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$

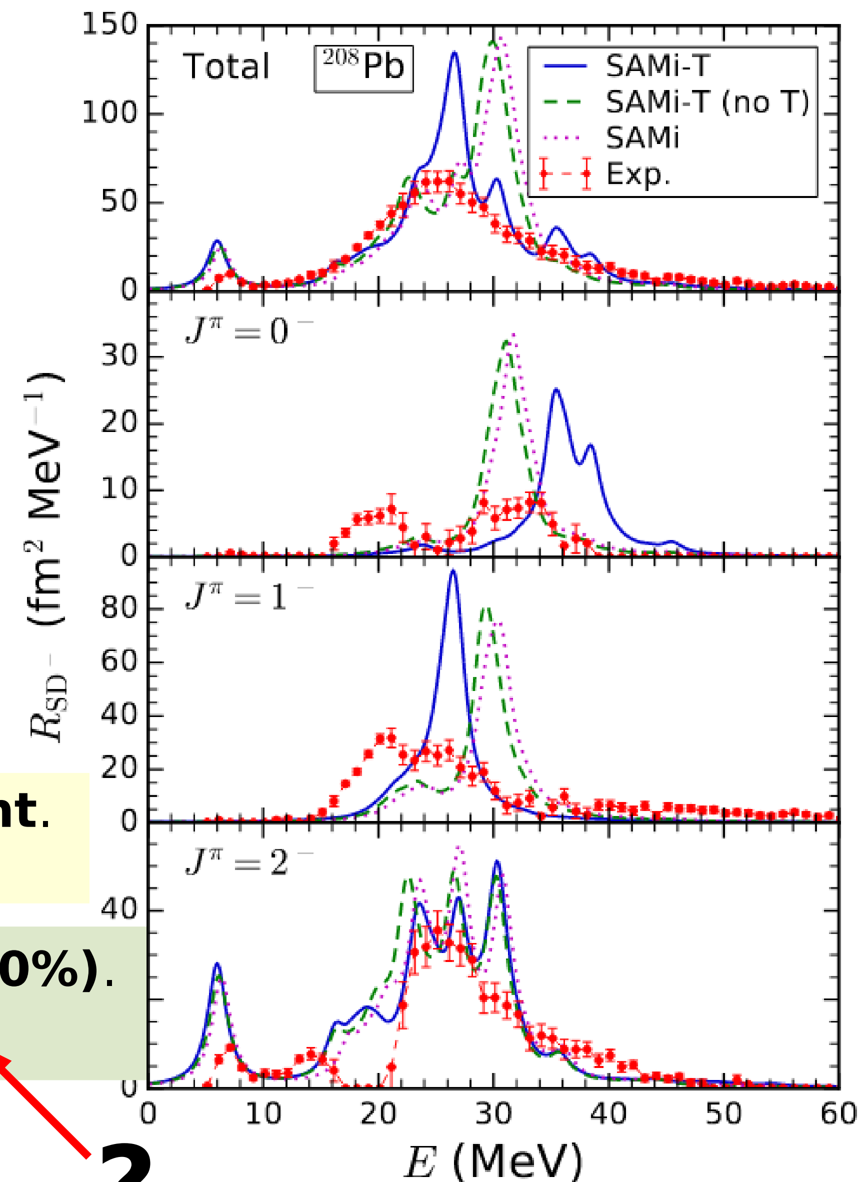
$$m_0(t_-) - m_0(t_+) = \frac{9}{4\pi}(N-Z)\langle r_p^2\rangle\left[1 + \frac{2N}{N-Z}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}} + \frac{N}{N-Z}\left(\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right)^2\right]$$

$$\approx \frac{9}{4\pi}(N-Z)\langle r_p^2\rangle\left(1 + \frac{2N}{N-Z}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right)$$

The only straight forward sum rule giving information on the skin!!

Spin Dipole Resonance and Δr_{np}
Difficult to measure?





→ ^{90}Zr **exp and theo sum rules in agreement**.
From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.04$ fm

→ ^{208}Pb **exp sum rule < theo sum rule (~20%)**.
From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.03$ fm

?

Shihang Shen (申时行), Gianluca Colò, and Xavier Roca-Maza
Phys. Rev. C **99**, 034322 – Published 20 March 2019

Summary from Progress in Particle and Nuclear Physics 101 (2018) 96–176

EoS par.	Observable	Range	Comments
ρ_0	$\langle r_{ch}^2 \rangle^{1/2}$	0.154–0.159	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
e_0	$M(N,Z)$	-16.2 to -15.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
K_0	$M(N,Z)$	220–245	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	ISGMR	220–260	From EDFs in closed shell nuclei [116]
	ISGMR	250–315	Blaizot's formula [Eq. (32)] [51]
	ISGMR	$\sim$200	EDF describing also open shell nuclei [118]
J	$M(N,Z)$	29-35.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	IVGDR	$\sim$24.1(8) + L/8	From EDF analysis [$S(\rho = 0.1\ \mathrm{fm}^{-3}) = 24.1(8)$ MeV] [273]
	PDS	30.2–33.8	From EDF analysis [370]
	PDS	31.0–33.6	From EDF analysis [371]
	α_D	$24.5(8) + 0.168(7)L$	From EDF analysis ^{208}Pb [96]
	α_D	30–35	From EDF analysis [179]
	IAS and Δr_{np}	30.2–33.7	From EDF analysis [325]
	AGDR	31.2–35.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	32–33	From EDF analysis [508]
	compilation	29.0–32.7	[106]
	compilation	30.7–32.5	[107]
	compilation	28.5–34.9	[3]
L	$M(N,Z)$	27–113	Most accurate EDFs on $M(N,Z)$ $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	ρ_n	40–110	proton-^{208}Pb scattering [24]
	ρ_n	0–60	π photoproduction (^{208}Pb) [181]
	ρ_n	30–80	antiprotonic at. (EDF analysis) [102,509]
	ρ_{weak}	>20	Parity violating scattering [27]
	PDS	32–54	From EDF analysis [370]
	PDS	49.1–80.5	From EDF analysis [371]
	α_D	20–66	From EDF analysis [179]
	IVGQR and ISGQR	19–55	From EDF analysis [101]
	IAS and Δr_{np}	35–75	From EDF analysis [325]
	AGDR	75.2–122.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	45.2–54.6	From EDF analysis [508]
	compilation	40.5–61.9	[106]
	compilation	42.4–75.4	[107]
	compilation	30.6–86.8	[3]

Some alternative compilations:

M. Oertel, M. Hempel, T. Klähn, S. Typel, Rev. Modern Phys. 89 (2017) 015007.

M.B. Tsang, et al., Phys. Rev. C 86 (2012) 015803.

Bao-An Li, Xiao Han, Phys. Lett. B 727 (1) (2013) 276–281.

Summary

with qualitative indication of accuracy needed to describe experiment (note that absolute values might be subject to systematics)

→ $\rho_0 \in$ [0.154,0.159] fm^{-3} → relative accuracy 2%

→ needed to describe experiment (Rch) ≤0.1%

→ $e_0 \in$ [15.6,16.2] MeV→ relative accuracy 4%

→ needed to describe experiment (B) ≤0.0001%

→ $K_0 \in$ [200,260] MeV→ relative accuracy 25%

→ needed to describe experiment (E_x^{GMR}) ≤7%

→ $J \in$ [30,35] MeV → relative accuracy 15%

→ needed to describe experiment (α) ≤15%

→ $L \in$ [20,120] MeV → relative accuracy 150%

→ needed to describe experiment (α) ≤50%

→ ...

Collaborators

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