Equation of State for neutron star merger simulations

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Seminar @ University of Milano-Bicocca

Milano March 2nd, 2023

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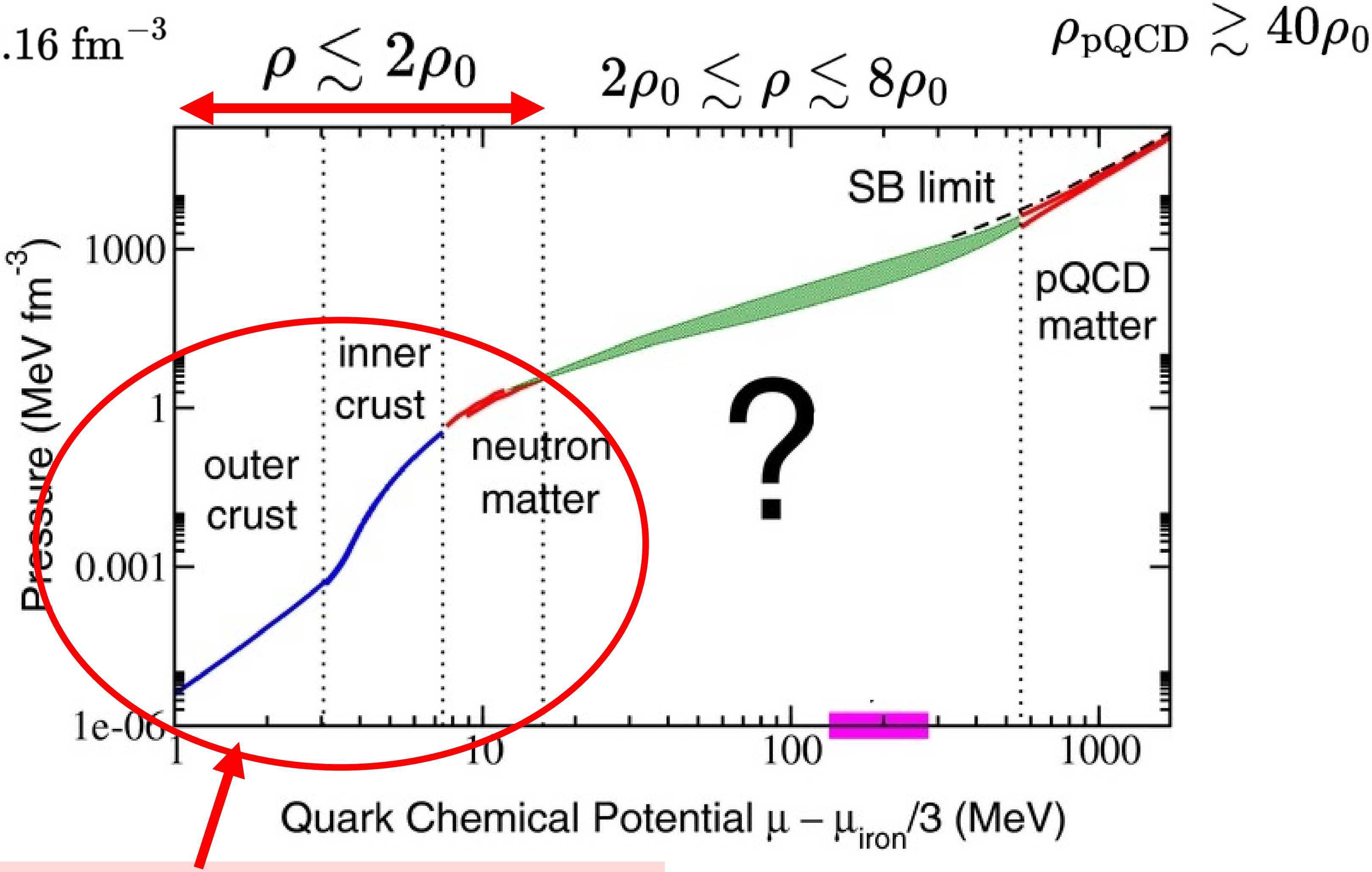
Neutron Star Equation of State

$\rho_0 = 0.16\ \mathrm{fm}^{-3}$



Low energy Nuclear Physics Domain

CONSTRAINING NEUTRON STAR MATTER WITH QUANTUM CHROMODYNAMICS

Aleksi Kurkela[1], Eduardo S. Fraga[2,3,4], Jürgen Schaffner-Bielich[2], and Aleksi Vuorinen[5]

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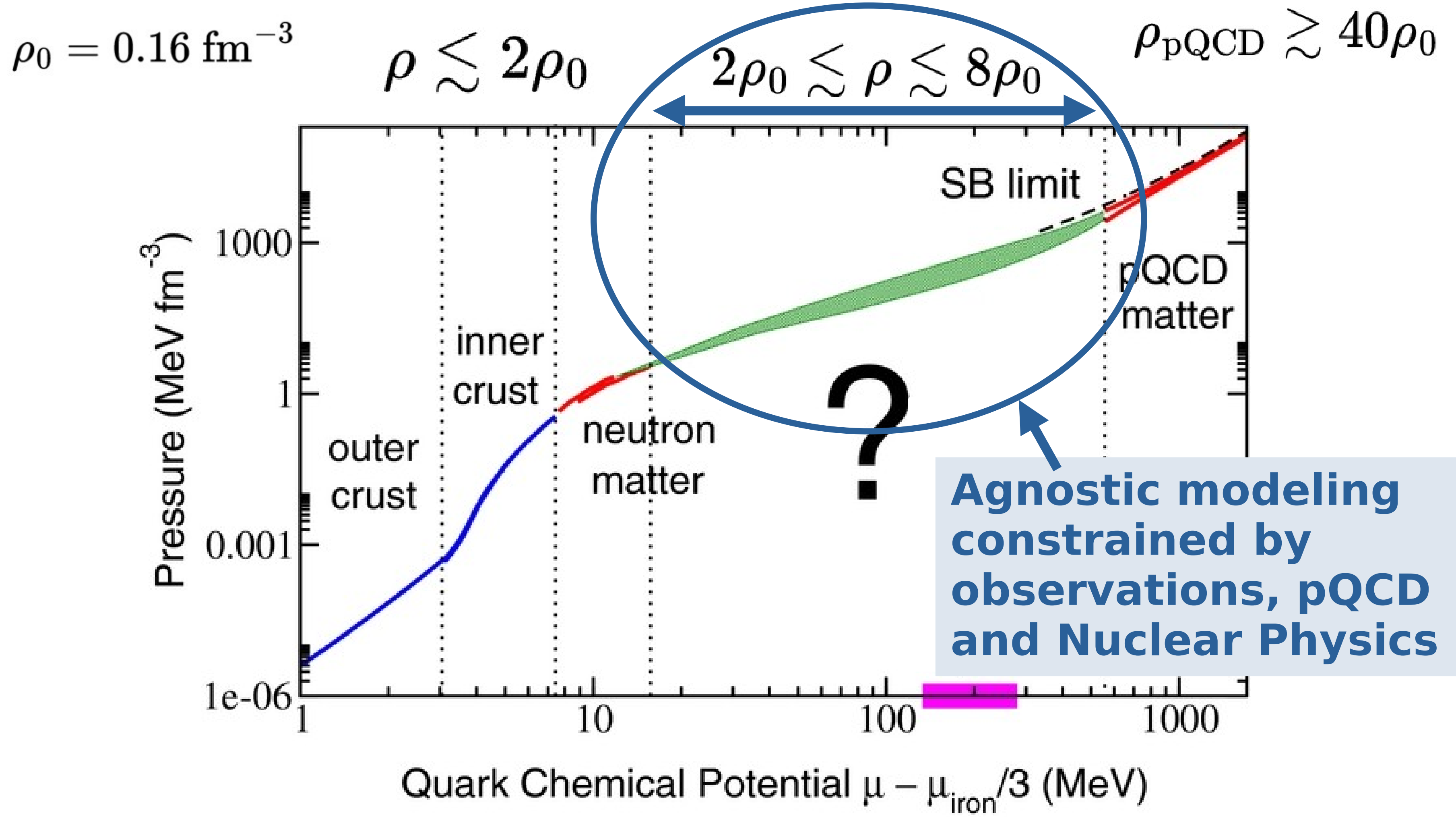
The Astrophysical Journal, Volume 789, Number 2

Citation Aleksi Kurkela *et al* 2014 *ApJ* **789** 127

Neutron Star Equation of State



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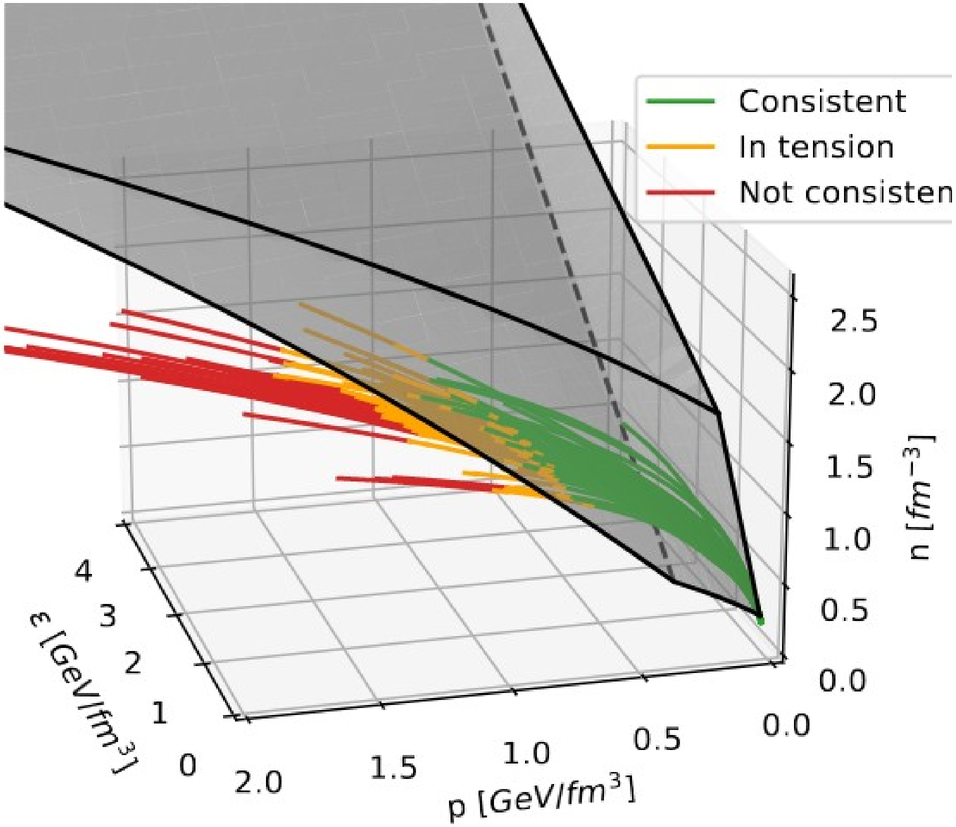
Neutron Star Equation of State

How Perturbative QCD Constrains the Equation of State at Neutron-Star Densities

Oleg Komoltsev and Aleksi Kurkela
Phys. Rev. Lett. **128**, 202701 – Published 18 May 2022

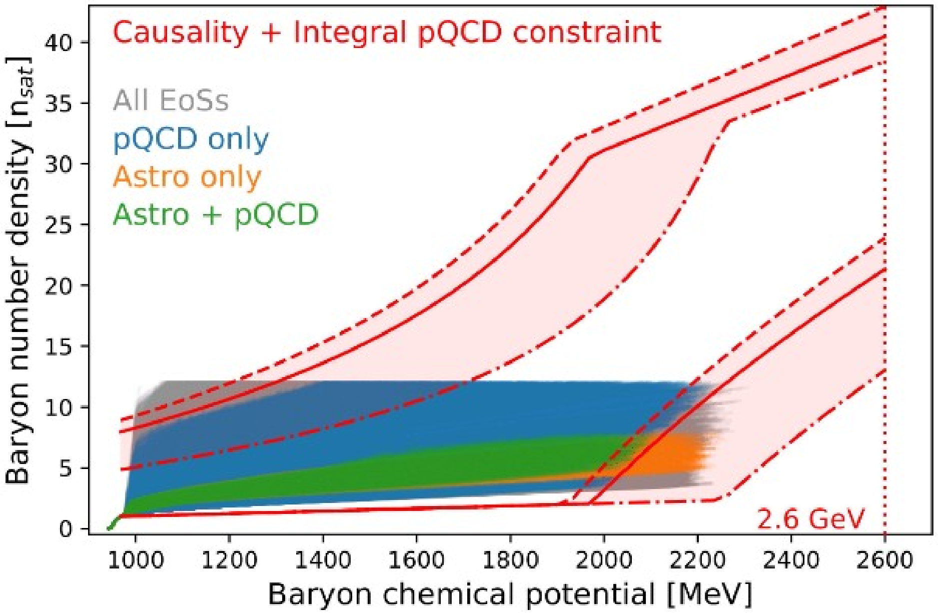


Constraints on **EoS from the CompOSE** database

Perturbative QCD and the Neutron Star Equation of State

Rahul Somasundaram, Ingo Tews, Jérôme Margueron

arXiv:2204.14039 [nucl-th]



Main constraints on EoS from astrophysical observations (Λ, NICER, Maximum mass, …)

Project:

Nuclear Physics can **potentially** put stringent and reliable **constraints** into a neutron star **EoS**

In this **presentation** I will **motivate** why we must do **better** at **low-densities**

PhD **Pietro Klausner** [UniMi]

In collaboration with:

Enrico Vigezzi [INFN]	Low denisty	$\rho<2\rho_0$
Francesca Gulminelli [UniCaen]	Densities	$\rho>2\rho_0$
Marco Antonelli [UniCaen]	Densities	$\rho>2\rho_0$
Bruno Giacomazzo [UniMiB]	NS merging	

Time

Nuclear Equation of State (EoS)

Main ingredient to build the neutron star EoS at low densities < 1-2ρ_0

Definition: the **energy** per **nucleon** ($e=E/A$ where $A=N+Z$) of an **uniform system of neutrons and protons** as a function of the **neutron** (ρ_n = N/V) and **proton** (ρ_p = Z/V) **densities**, at **zero temperature**, **unpolarized**, assuming **isospin symmetry** and **neglecting Coulomb** effects among protons.

Why???

→ **Zero temperature:** room temperature 10^2K→**10^{-8} MeV**. **Separation energy** in stable nuclei (equivalent to ionization energy in atoms) is of **several MeV**.

→ **Unpolarized:** energy favours **couples** of neutrons and protons **occupying the same state but with opposite spins** (equivalent to electrons in atoms)

→ **Isospin symmetry:** neutron-neutron, proton-proton and neutron-proton **nuclear interaction** are very **similar** among them. **Masses** of neutrons and protons are almost **degenerate**. Hence neutrons and protons can be thought as **two states** of the **same particle** with different isobaric spin or **isospin** (in analogy with spin): the **nucleon.**

→ **No Coulomb:** **idealized uniform system (focus on strong interaction)**. Real systems are finite and frequently electrically neutral so no problems (divergences) in adding Coulomb.

[Besides that, the strong interaction at the typical scale of a nucleus is much stronger than the Coulomb interaction and the Coulomb energy (*) per particle of an infinite system of protons would be infinite.]

$$(*)\text{Coulomb interaction infinite range!!} \rightarrow \frac{E_C}{A} \sim \frac{Z(Z-1)}{A^{4/3}} \sim Z^{2/3}$$

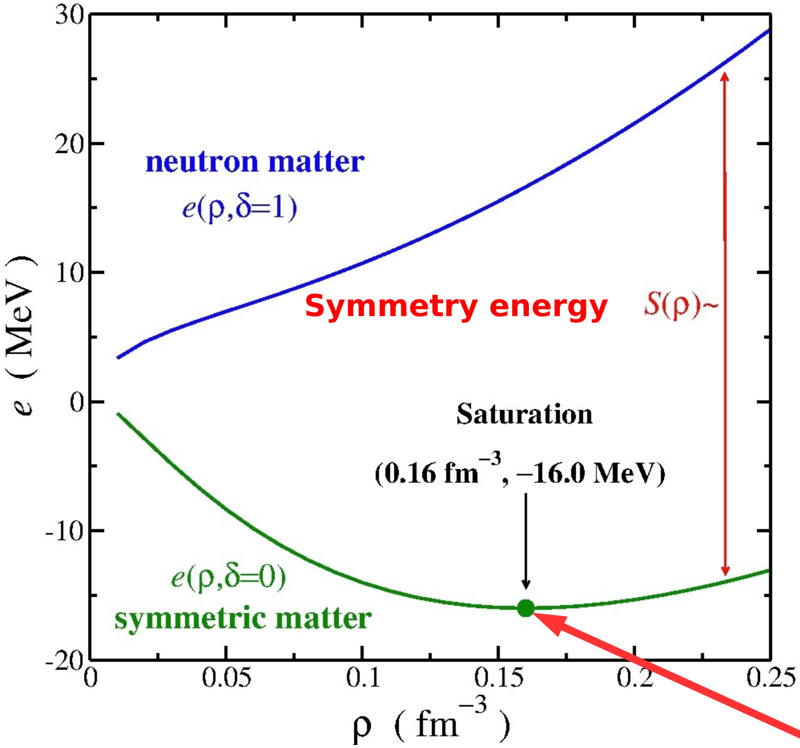
Nuclear Equation of State (EoS)

It is **convenient** to write the **energy per nucleon** (e) as a **function** of the **total density** [$\rho=\rho_n+\rho_p$] and their **relative difference** [$\delta=(\rho_n - \rho_p)/\rho$].

→ Due to isospin symmetry only even powers of δ will appear

→ Stable nuclei tend to show small values of δ

Taylor expansion for δ→ 0: $$e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4]$$



It is customary to also **expand** e(ρ,0) and S(ρ) around nuclear **saturation density** $\rho_0 \sim 0.16$ fm^{-3}

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0 x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

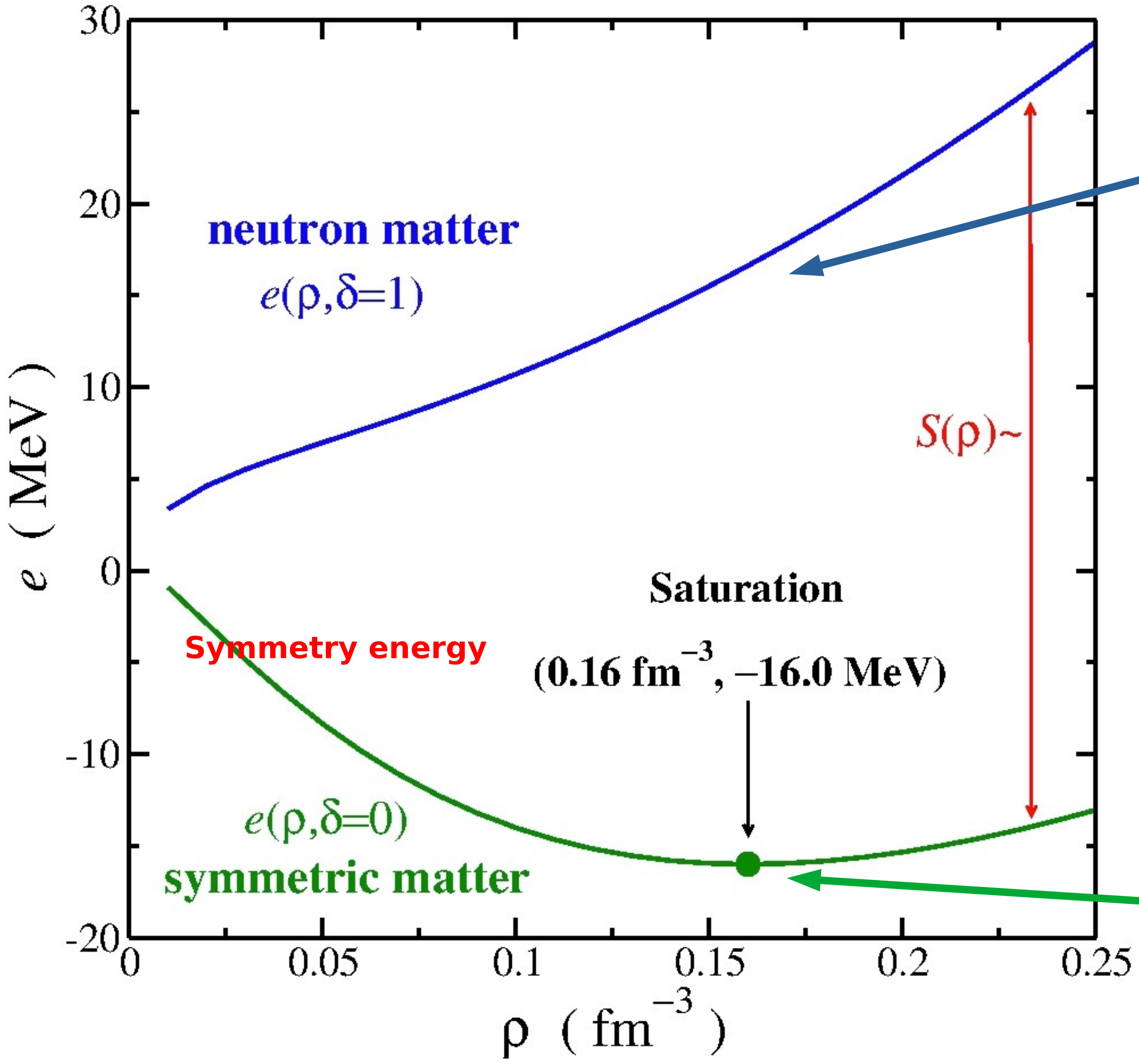
K_0 → how **compressible** is symmetric matter at ρ_0

J (a_A) → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L (a_s) → **neutron pressure** in neutron matter at ρ_0

$$P(\rho=\rho_0,\delta=0) = 0 \text{ MeV fm}^{-3}$$

Nuclear Equation of State (EoS)



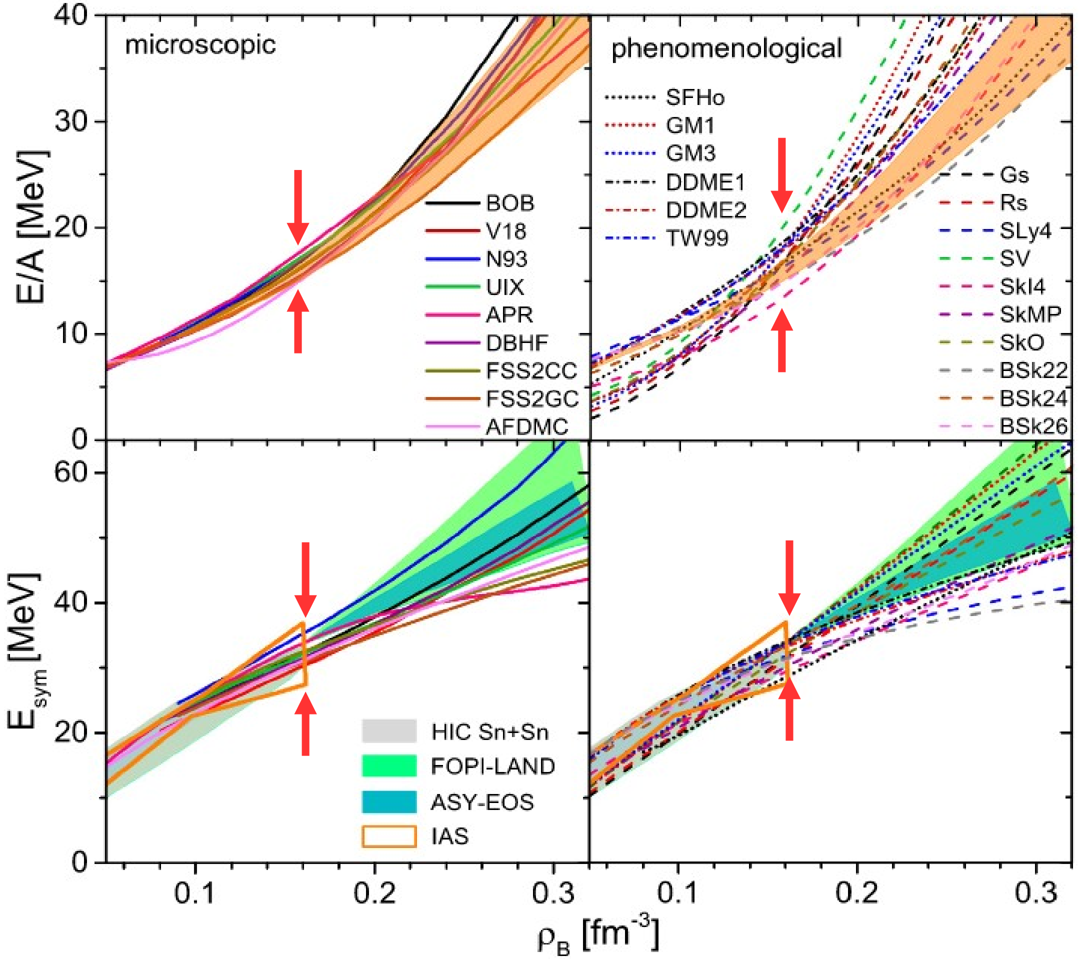
Neutron matter:
Essential input in building a neutron star EoS

Symmetry energy allows to "connect" nuclei with neutron stars (there exist only few reliable nuclear obervables informing us)

Symmetric matter:
Nuclear observables essentially inform us around this region

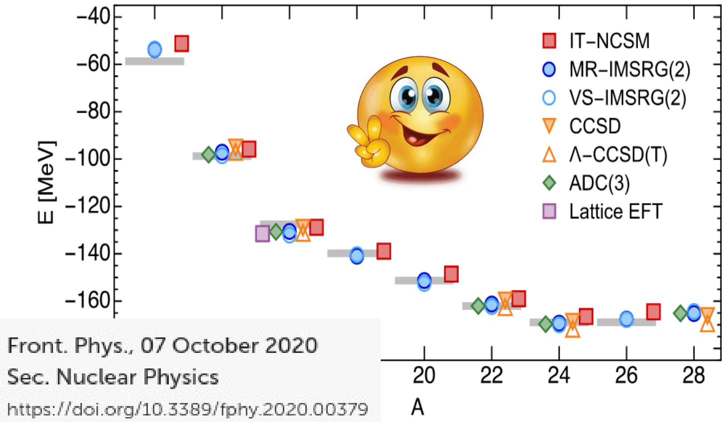
EoS from current nuclear models

Micorscopic and phenomenological models constrainted by different data display similar discrepances on the EoS



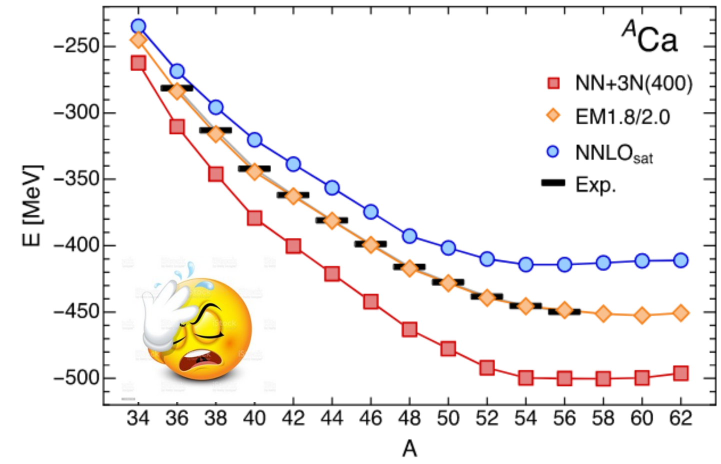
Symmetry energy **not well constrained** and specially **important** for extrapolations to very **dense neutron matter**

Many-body methods have been shown to agree



Front. Phys., 07 October 2020
Sec. Nuclear Physics
https://doi.org/10.3389/fphy.2020.00379

Main source of uncertainty is the nuclear Hamiltonian



Progress in Particle and Nuclear Physics 120 (2021) 103879

Light orange bands from χEFT from Drischler C., et al. Phys. Rev. Lett., 125 202702 (2020).

Summary: nuclear models performance

with qualitative indication of accuracy needed to describe experiment (note that absolute values might be subject to systematics)

- → $\rho_0 \in$ [0.154,0.159] fm^{-3} → relative accuracy 2%
  - → needed to describe experiment (Rch) ≤0.1%
- → $e_0 \in$ [15.6,16.2] MeV→ relative accuracy 4%
  - → needed to describe experiment (B) ≤0.0001%
- → $K_0 \in$ [220,260] MeV→ relative accuracy 17%
  - → needed to describe experiment (E_x^{GMR}) ≤7%
- → J ∈ [30,35] MeV → relative accuracy 15%
  - → needed to describe experiment (α) ≤15%
- → L ∈ [20,120] MeV → relative accuracy 150%
  - → needed to describe experiment (α) ≤50%
- → ...

L→Neutron pressure!

Progress in Particle and Nuclear Physics
Volume 101, July 2018, Pages 96-176

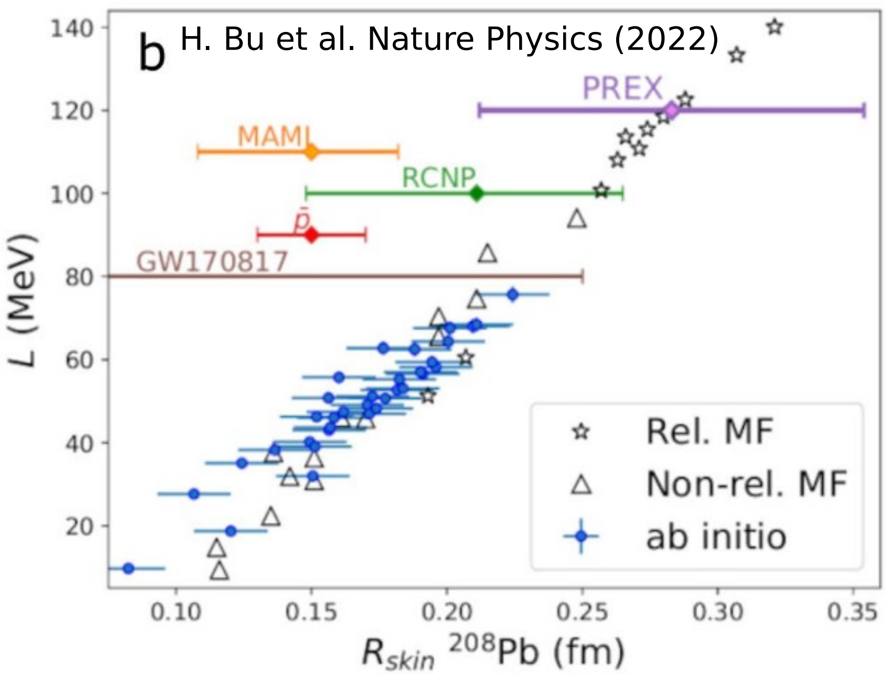
What can we learn from laboratory observables on the symmetry energy around ρ_0?

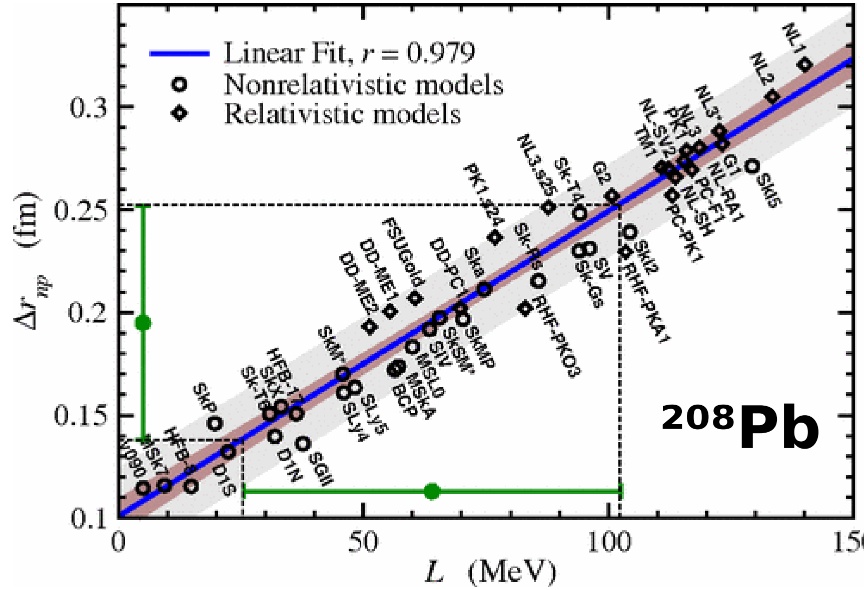
Neutron skin is a good proxy to L

B. Alex Brown Phys. Rev. Lett. 85, 5296 (2000)

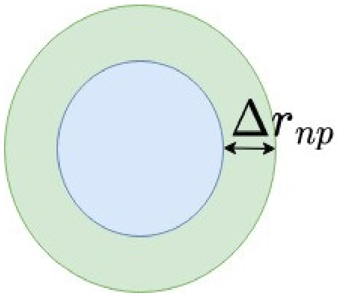
The **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in a heavy neutron rich nucleus is related to the **neutron pressure** ($\delta=1$) around ρ_0 (L).

$$P(\rho_0,\delta)=\rho_0^2\frac{\partial e(\rho,\delta)}{\partial\rho}\bigg|_{\rho=\rho_0}=\frac{1}{3}\rho_0\delta^2 L$$





Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)



→ From the Droplet Model:

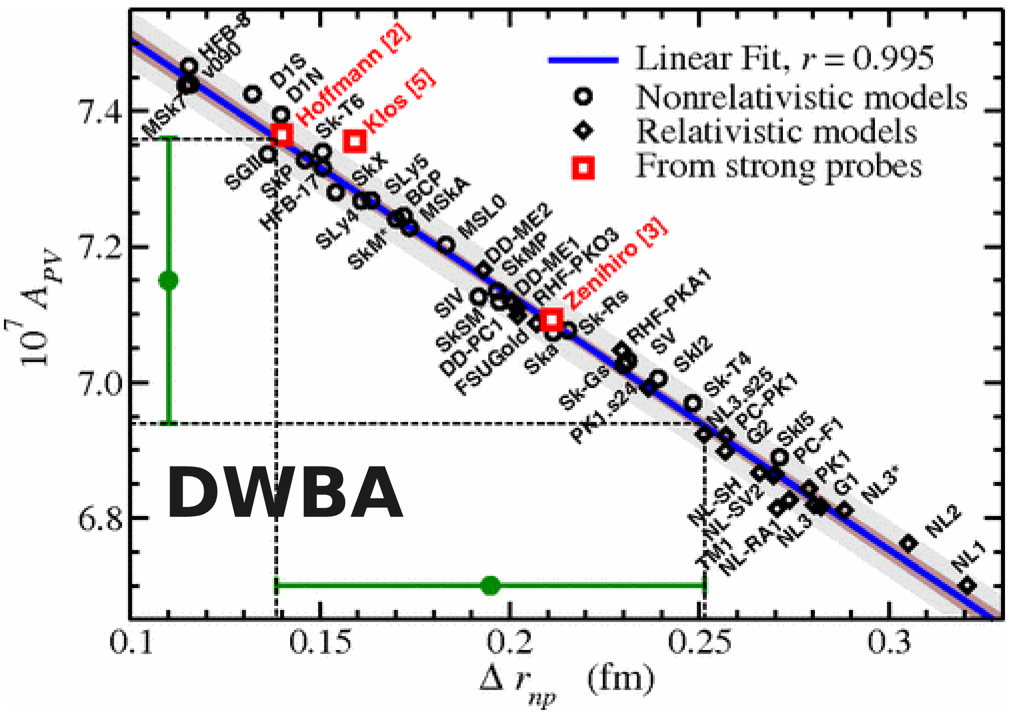
$$\Delta r_{np}\approx\frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

But how can we measure the neutron skin thickness?

Polarized electron-Nucleus scattering:

→ In good approximation, **the weak interaction** probes the **neutron distribution** in nuclei while **Coulomb interaction** probes the **proton distribution**

→ **Different experimental efforts @ Jlab (USA) & MAMI (Germany)**



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

→ **Electrons** interact by **exchanging** a **γ** (couples to **p**) or a $\mathbf{Z_0}$ boson (couples to **n**)

→ **Ultra-relativistic electrons**,depending on their helicity (±), will interact with the nucleus seeing a slightly different potential: **Coulomb** ± **Weak**

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

→ Main **unknown** is $\boldsymbol{\rho_n}$

→ In **PWBA** for small momentum transfer **q**:

$$A_{pv} = \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 r_p^2}{3F_p(q)}\right)\Delta r_{np}$$

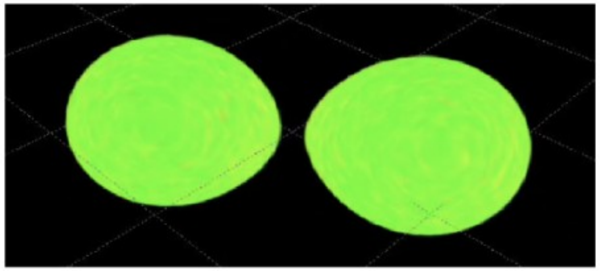
What else?
Nuclear response to external probes

→ **Analogy:** How easy a **nucleus** polarize due to an external Coulomb field or a **neutron star** deforms due to an external gravitational field can be estimated

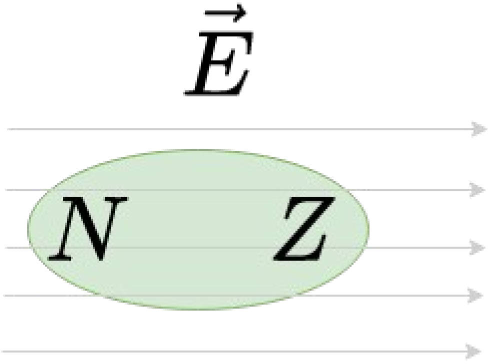
$$\sum_{\nu} \frac{|\langle \nu | \mathcal{O} | 0 \rangle|^2}{E_\nu - E_0}$$

Excitation operator → Gravitational Field → **tidal deformability**

Excitation operator → Electric Field → **nuclear polarizability**



Gravitational tidal field distorts shapes of neutron stars just before they merge.

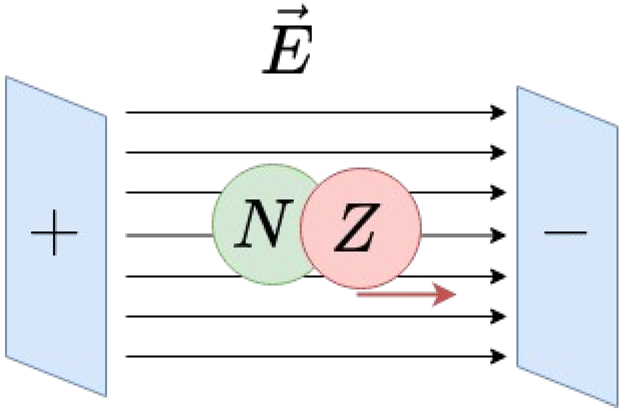


Electric field distorts the nucleus (neutrons do not “feel” ***E*** but strongly bound to protons)

Dipole polarizability

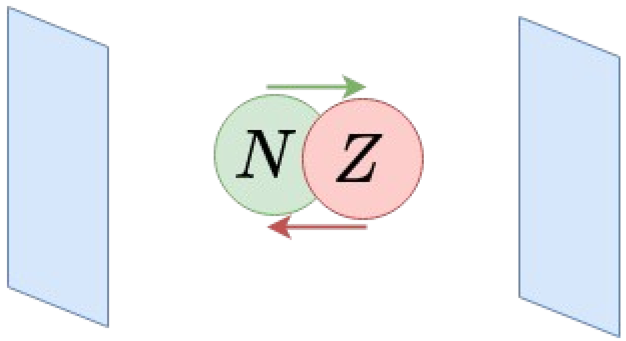
The **electric polarizability** measures **tendency** of the nuclear **charge distribution** to be **distorted**

$$\alpha = \frac{\text{electric dipole moment}}{\text{external electric field applied}}$$



Polarizability is **proportional** to the photoabsorption cross section:

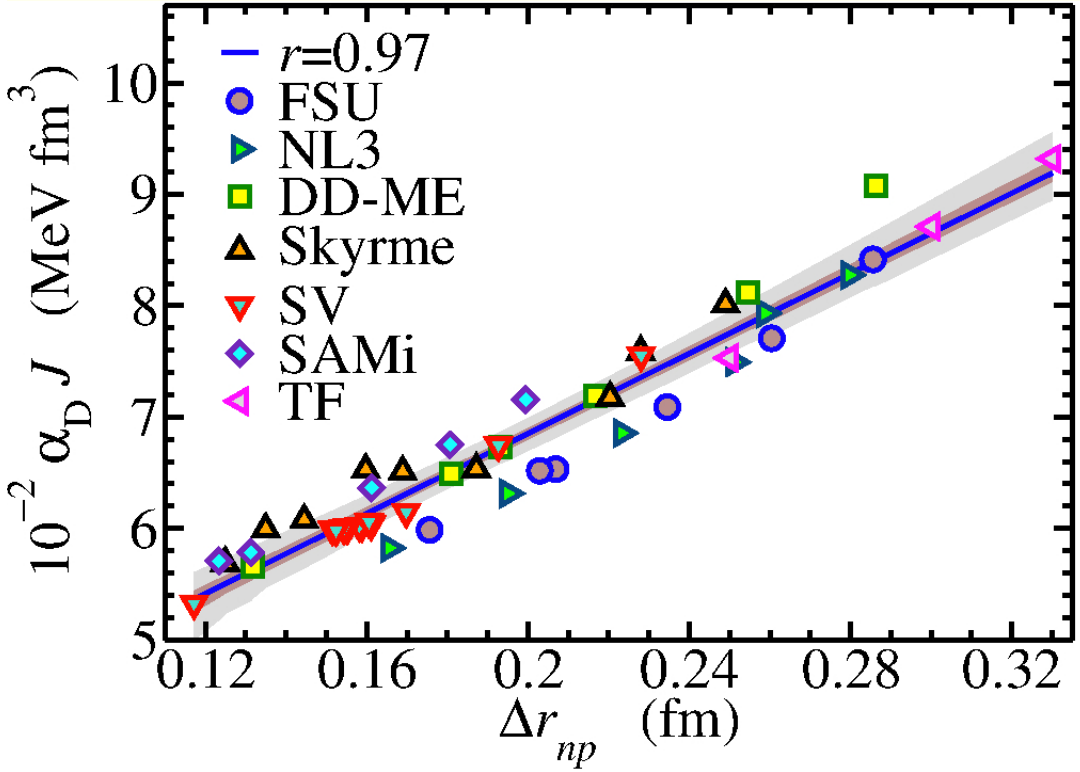
$$\alpha_D = \frac{\hbar c}{2\pi^2 e^2} \int \frac{\sigma_{\text{abs}}}{\omega^2} d\omega,$$



How easy is to separate neutrons from protons? Symmetry energy will tell (restoring force between neutrons and protons).

Dipole polarizability and neutron skin

From **state-of-the-art nuclear models** one finds a **correlation** between **α$_D$**, **the symmetry energy** (J) and the **neutron skin thickness**



Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz Phys. Rev. C 88, 024316 (2013)

→ This relation is also supported by simple models.

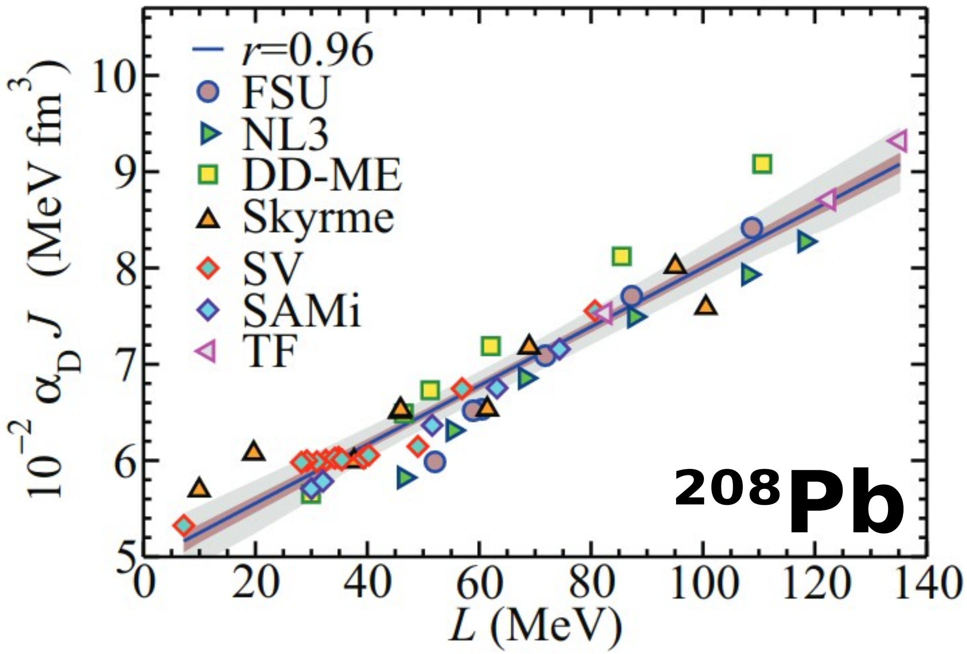
→ For guidance assuming the **Droplet Model**, one would find:

$$\alpha_D \approx \frac{\pi e^2}{54}\frac{\langle r^2\rangle}{J}A\left(1+\frac{5}{2}\frac{\Delta r_{np}-\Delta r_{np}^{\rm surf}-\Delta r_{np}^{\rm Coul}}{\langle r^2\rangle^{1/2}(I-I_{\rm Coul})}\right)$$

Polarizability increases with the mass (for the dipole $A^{5/3}$, for the quadrupole $A^{7/3}$ and so on …) and it **sets a relation between the EoS parameters J and L**

Dipole polarizability, J and L

Determination of the J vs L relation from experimental data according to EDFs



$$J = 25.0(2) + 0.19(2)L \quad \text{for } ^{68}\text{Ni},$$
$$J = 25.4(1.1) + 0.17(1)L \quad \text{for } ^{120}\text{Sn},$$
$$J = 24.5(8) + 0.17(1)L \quad \text{for } ^{208}\text{Pb}.$$

Assuming Taylor expansion around ρ_0:

$$S(\langle\rho\rangle) \approx J - L\frac{\rho_0 - \langle\rho\rangle}{3\rho_0} \rightarrow J \approx S(\langle\rho\rangle) + \frac{\rho_0 - \langle\rho\rangle}{3\rho_0} L$$

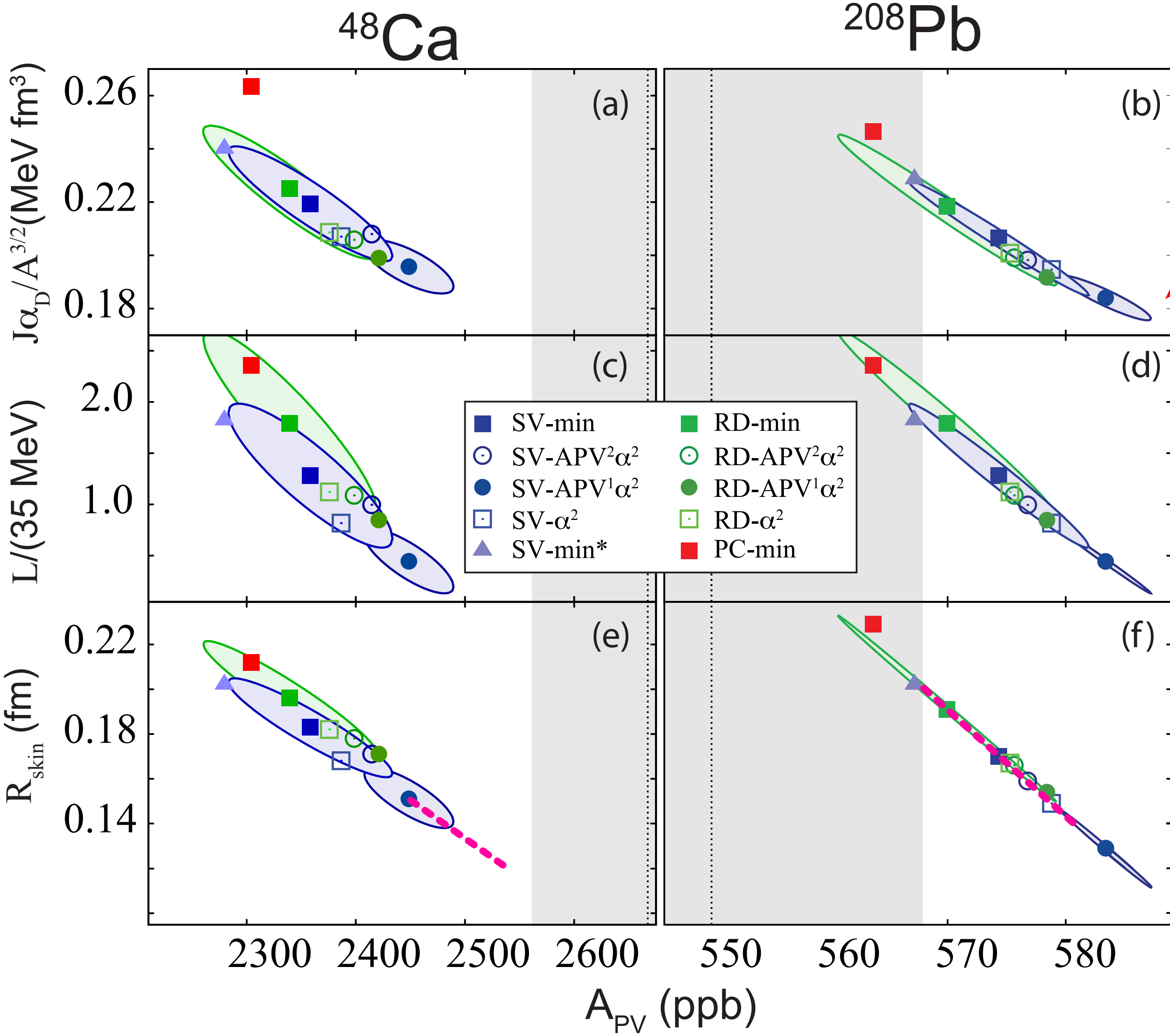
one can qualitatively understand the result!!

$S(\langle\rho\rangle \approx 0.08\ \text{fm}^{-3}) \approx 25\ \text{MeV}$

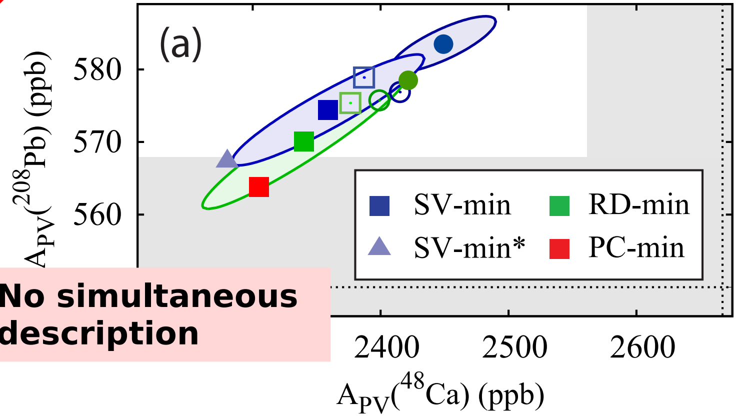
X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013

X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

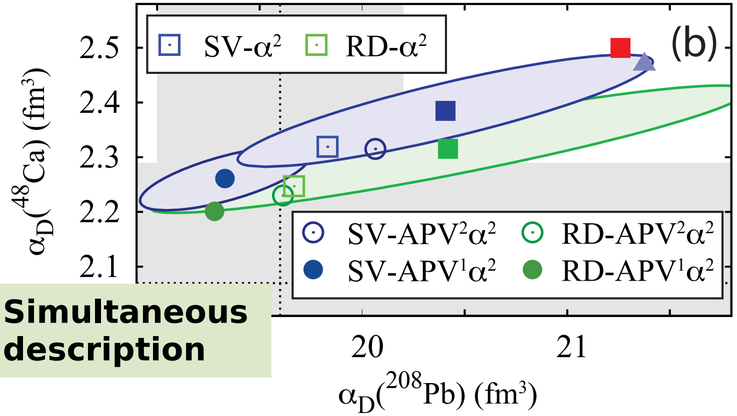
α_D and A_{PV} in ^{48}Ca and ^{208}Pb



Theoretical (**EDFs** and **ab initio**) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca



No simultaneous description

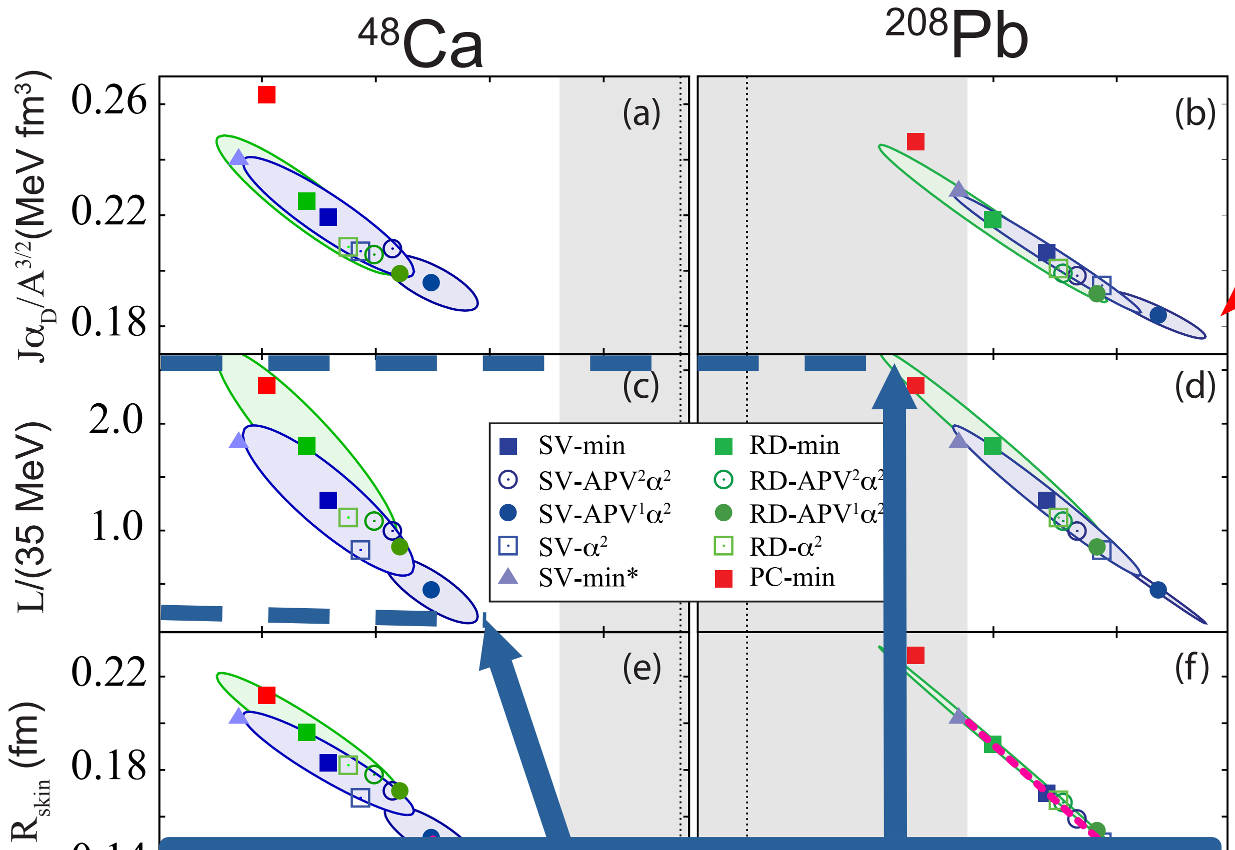


Simultaneous description

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz PRL127, 232501 (2021) and PRL 129, 232501 (2022)

Magenta dashed lines from Δr_{np} in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

α_D and A_{PV} in ^{48}Ca and ^{208}Pb



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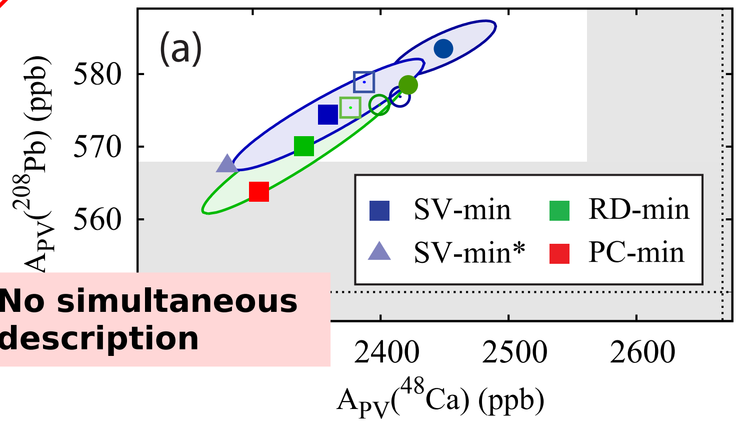

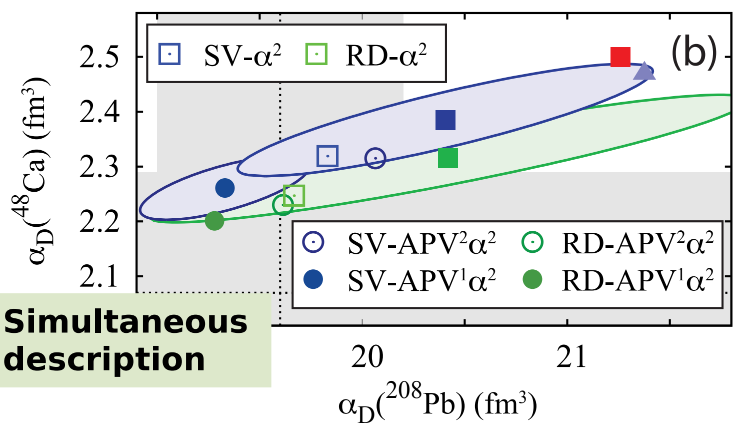

Small neutron pressure (*L*) or large neutron pressure?

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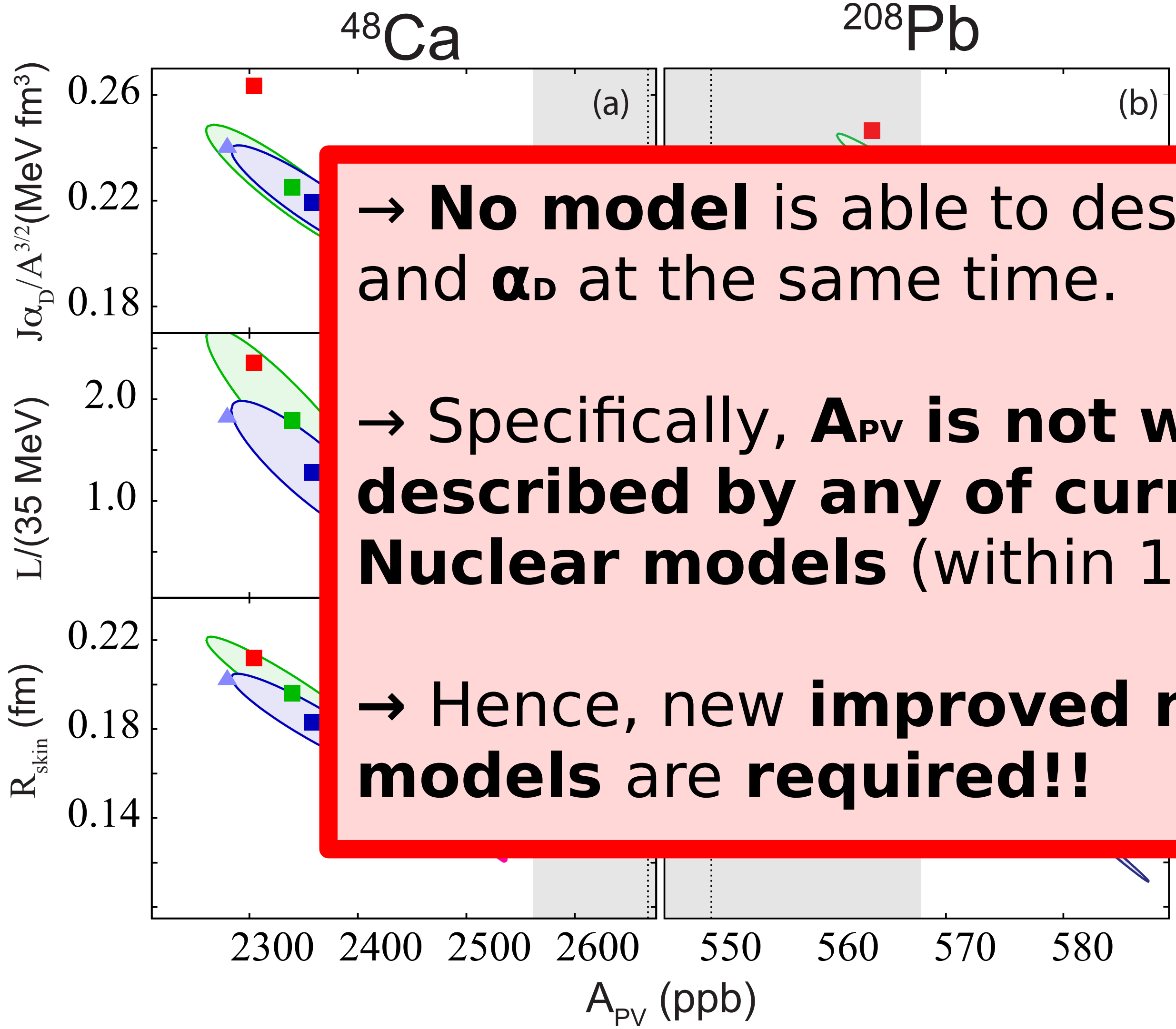
α_D and A_{PV} in ^{48}Ca and ^{208}Pb



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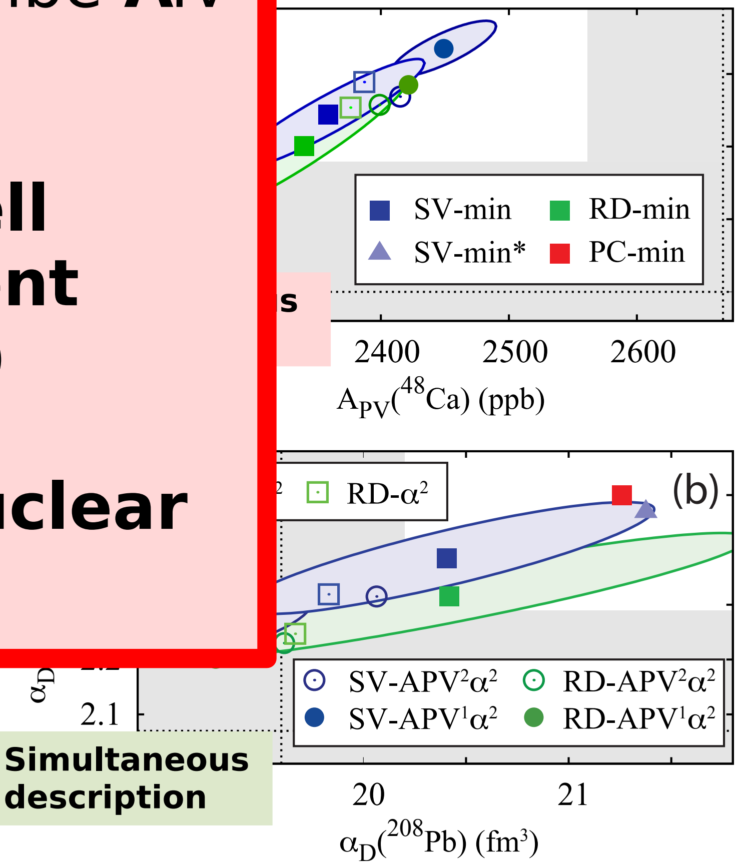

Simultaneous description

→ **No model** is able to describe $\mathbf{A_{PV}}$ and $\boldsymbol{\alpha_D}$ at the same time.

→ Specifically, $\mathbf{A_{PV}}$ **is not well described by any of current Nuclear models** (within 1σ)

→ Hence, new **improved nuclear models** are **required!!**

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz PRL127, 232501 (2021) and PRL 129, 232501 (2022)

Magenta dashed lines from Δr_{np} in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Conclusion:

→ **Dense neutron matter** ($\rho<2\rho_0$) must be described by **nuclear models**

→ Current **nuclear models do not reproduce A_{PV}** and at the same time other relevant observables

→ **Theory must provide new improved models** for reliable applications in **nuclear astrophysics.**

Collaborators:

→ Gianluca **Colò** (University of Milan)

→ Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)

→ Jorge **Piekarewicz** (Florida State University)

→ Nils **Paar** & Dario **Vretenar** (University of Zagreb)

→ Bijay K. **Agrawal** (Saha Institute of Nuclear Physics)

→ P.-G. **Reinhard** (University of Erlangen-Nürnberg)

→ Witold **Nazarewicz** (FRIB and Michigan State University)