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MOdeling
Nuclear
STructure and
REactions

The electric dipole polarizability and the nuclear Equation of State

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R3B Colloquium

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UNIVERSITAT DE
BARCELONA



Definitions, phenomenology & simple models

The Giant Dipole Resonance

→ Imagine to apply an uniform **electric field** to a nucleus in its **ground state**

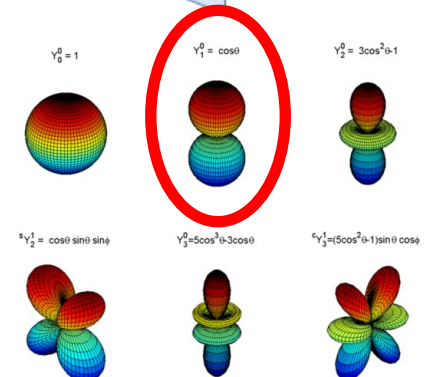
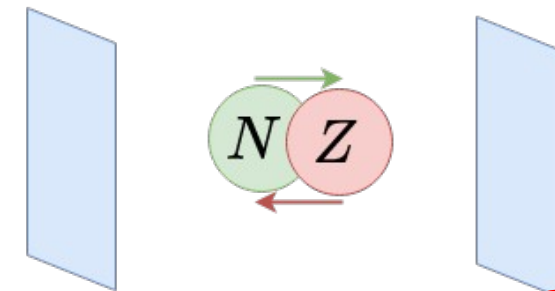
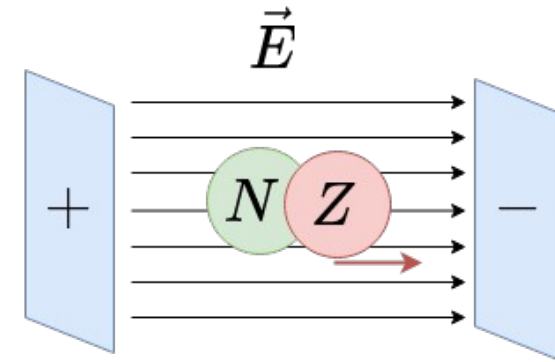
→ **Protons** will start **moving** to the right (**translation** due to **Coulomb force**)

→ At (essentially) the same time, **neutrons will follow protons (nuclear force)** and start to **oscillate**.

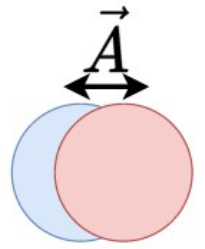
→ As a result one finds a translational motion of the whole nucleus plus an **oscillatory motion of neutrons against protons**. That is, $\rho_n(r,t) - \rho_p(r,t)$ oscillates (=isovector motion)

→ We are interested on the **oscillation frequency** since it must depend on the **attraction between neutrons and protons (restoring force)**

→ **Different multipolarities can contribute to this motion**, the **dominant** one will be the **dipole**



The Giant Dipole Resonance



→ The **Isovector Giant Dipole Resonance** was the first resonance measured in the atomic nucleus (**photo-absorption** experiments)

→ The **cross section** for the **excitation** of the nucleus to a **final state** $|\nu\rangle$ with energy E_ν from the **ground state** $|0\rangle$ with energy E_0 by a **photon** at a given energy E can be written as

Dominated by dipole $\sigma_\nu(E) = 4\pi^2\alpha(E_\nu - E_0)|\langle\nu|F_{\text{dipole}}|0\rangle|^2\delta(E - E_\nu + E_0)$ Convenient operator
[$\sim r Y_{10}(\Omega)$]
produces dipole
transitions and
subtract CM motion

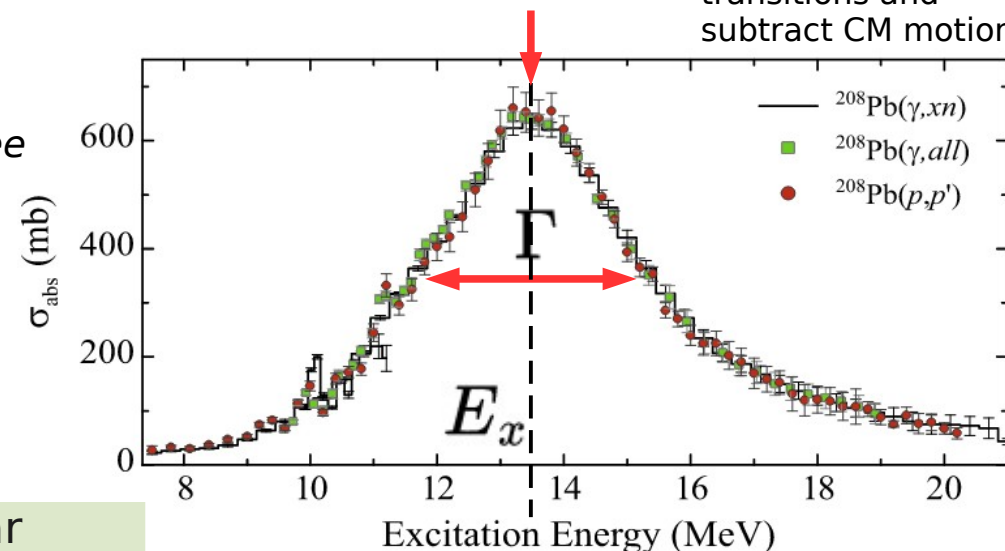
→ The **total cross-section** will be

$$\sigma_{\gamma\text{-abs}} = 4\pi^2\alpha \sum_{\nu} (E_\nu - E_0) |\langle\nu|F_{\text{dipole}}|0\rangle|^2 \quad (\text{EWSR see later})$$

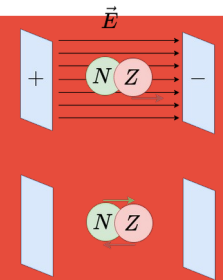
$$S(E) \equiv \sum_{\nu} |\langle\nu|F|0\rangle|^2 \delta(E - E_\nu + E_0)$$

where **S(E)** is the so called **Strength function**

S(E) is used to **characterize** the nuclear **response** (experimentally, it is commonly parametrized as a **Lorentzian function** with energy E_x and width Γ)



Electric Dipole Polarizability



The **electric dipole polarizability** measures the **tendency** of the nuclear **charge distribution** to be **distorted**

Macroscopically

$$\alpha = \frac{\text{electric dipole moment}}{\text{external electric field applied}}$$

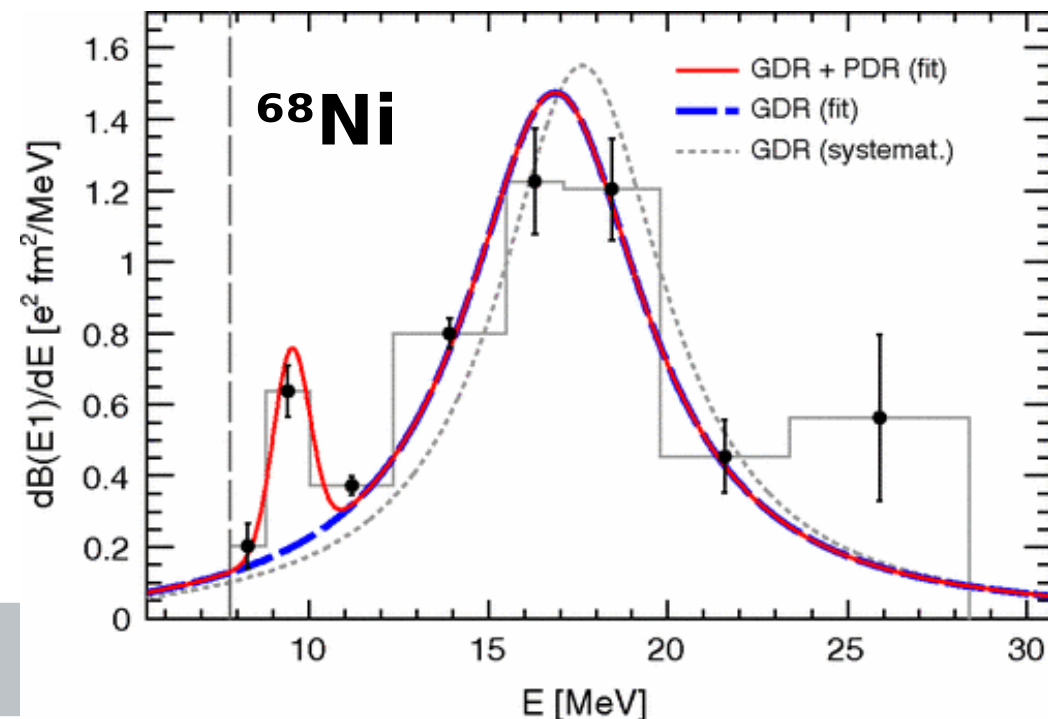
Microscopically, it relates with the **photo-absorption cross-section**

$$\alpha_D = \frac{\hbar c}{2\pi^2 e^2} \int_0^\infty dE \frac{\sigma_{\gamma-abs}}{E^2}$$

Integral from 0 to “infinity” → Full nuclear response known

In stable nuclei it has been measured, e.g., using **polarized** proton scattering at **very forward angles** (**dominated by E1** and **M1 well separated**)

In **unstable nuclei**, such as ^{68}Ni , it has been measured using **Coulomb excitation in inverse kinematics** (using the R3B-LAND setup)



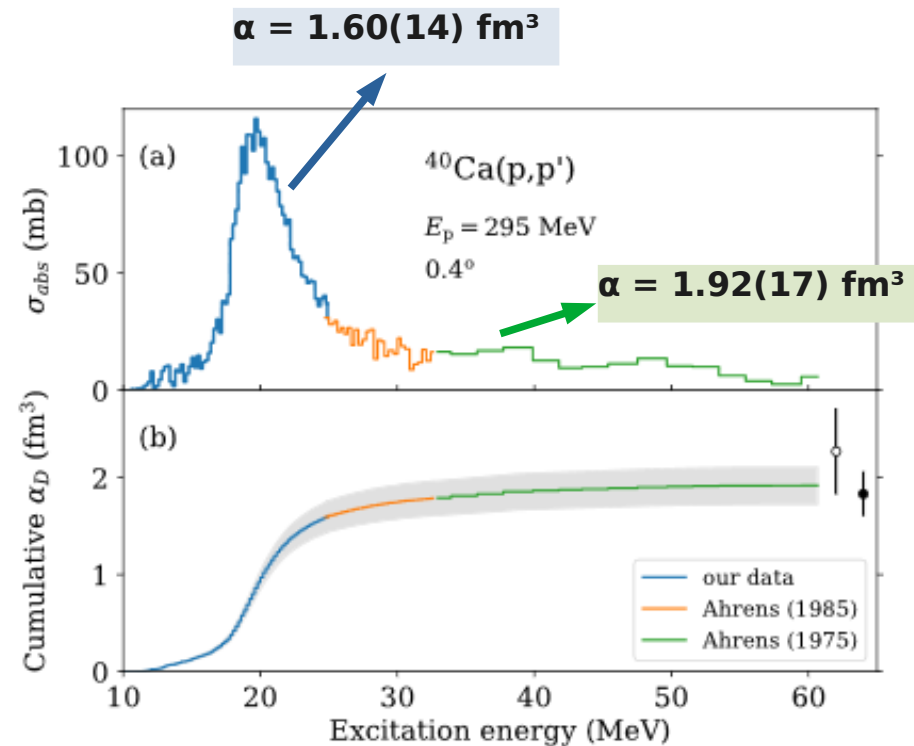
Electric Dipole Polarizability

$$\alpha_D = \frac{\hbar c}{2\pi^2 e^2} \int_0^\infty dE \frac{\sigma_{\gamma-abs}}{E^2}$$

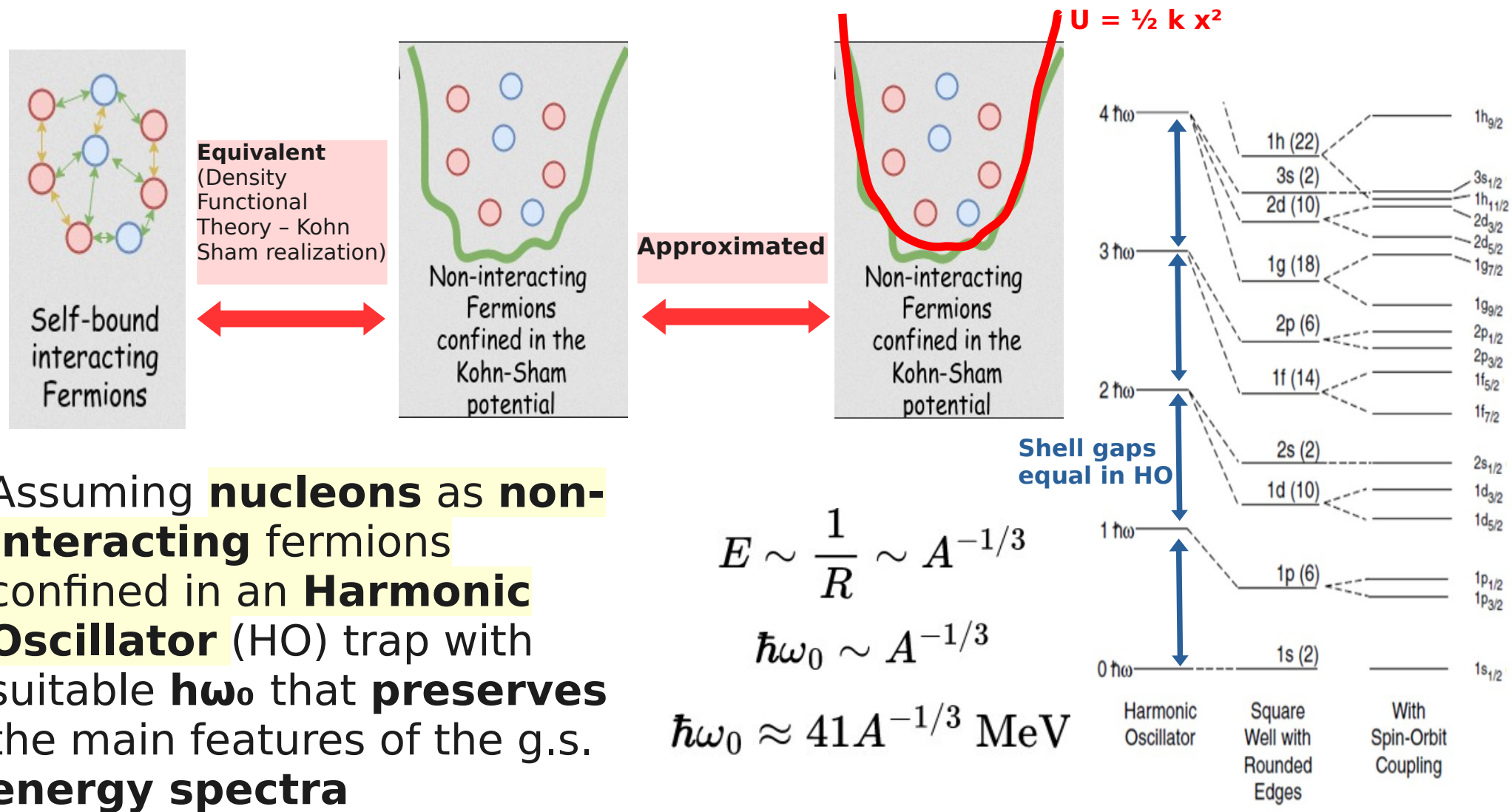
**Integral from 0 to “infinity”
→ Full nuclear response
must be known**

→ **High energy tail** of the photo-absorption cross section need to be **measured, extrapolated** or taken from **theory**. It is, nevertheless, a **small contribution due to $1/E^2$**

→ Discrete **states below the neutron separation energy** and **pygmy states** need to be **measured**. They constitute a **large contribution due to $1/E^2$**



Giant Resonances from theory: Harmonic oscillator



Giant Resonances from theory: Harmonic oscillator

The **HO Hamiltonian**, in terms of the **conjugate variables** α (or r position) and $d\alpha/dt$ (or v velocity) and the ($C_0 \leftrightarrow m\omega^2$) and ($B_0 \leftrightarrow m$) parameters, could be written as (Bohr&Mottelson):

$$\mathcal{H}_0 = \frac{1}{2}B_0\dot{\alpha}^2 + \frac{1}{2}C_0\alpha^2 \rightarrow E_0 = \hbar \left(\frac{C_0}{B_0} \right)^{1/2} \longleftrightarrow E = \left(\frac{1}{B_0} \frac{\partial^2 U}{\partial \alpha^2} \right)^{1/2}$$

Assume harmonic perturbation (restoring force)

$$F = -\kappa\alpha \rightarrow V = \frac{1}{2}\kappa\alpha^2$$

$$\mathcal{H} = \mathcal{H}_0 + V \rightarrow \mathcal{H} = \frac{1}{2}B_0\dot{\alpha}^2 + \frac{1}{2}(C_0 + \kappa)\alpha^2$$

$$E = E_0 \left(1 + \frac{\kappa}{C_0} \right)^{1/2}$$

Restoring force

Giant Resonances from theory: Harmonic oscillator

Generalized coordinate α will change with $\delta = (\rho_n - \rho_p) / (\rho_n + \rho_p)$ in an **isovector oscillation** of **neutrons against protons**. Hence,

$$\mathcal{H}_0 = \frac{1}{2} B_0 \dot{\alpha}^2 + \frac{1}{2} C_0 \alpha^2 \rightarrow E_x^2 \propto \frac{\partial^2 e}{\partial \delta^2}$$

Where $\mathbf{e} = \mathbf{e}(\rho_n, \rho_p, \dots)$ represents the **potential energy** of the system as a function of its variables. Assuming the **local density approximation** is valid:

It is **convenient** to write the **energy per nucleon** (e) as a **function** of the **total density** [$\rho = \rho_n + \rho_p$] and their **relative difference** [$\delta = (\rho_n - \rho_p) / \rho$].

→ Due to isospin symmetry only even powers of δ will appear

→ Stable nuclei have small values of δ

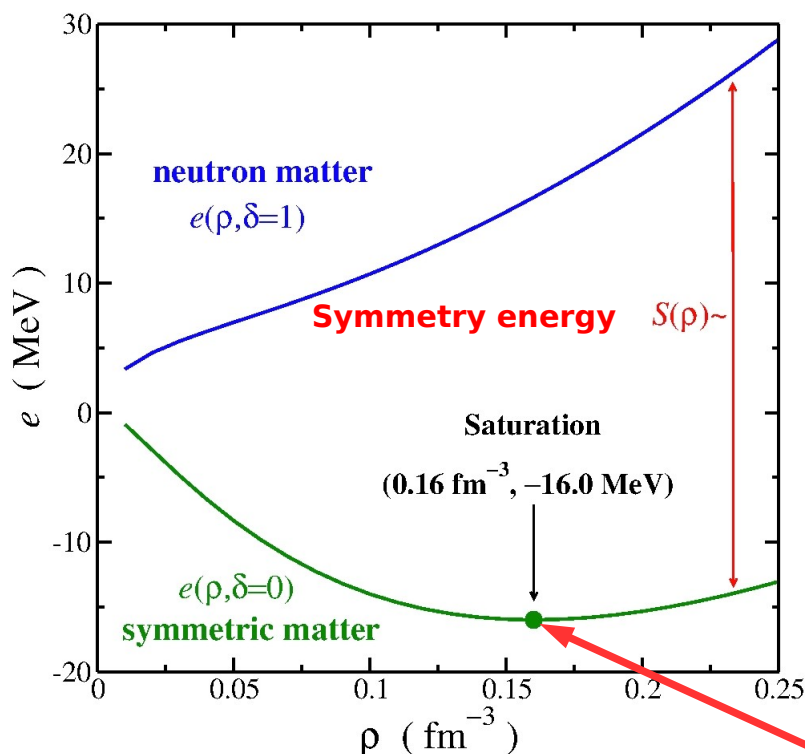
Taylor expansion for $\delta \rightarrow 0$: $e(\rho, \delta) = e(\rho, 0) + S(\rho) \delta^2 + \mathcal{O}[\delta^4]$

$$E_x^2 \propto \left. \frac{\partial^2 e}{\partial \delta^2} \right|_{\rho = \rho_{\text{eq.}}} = S(\rho_{\text{eq.}})$$

Symmetry energy

What is the symmetry energy? Nuclear Equation of State (EoS)

$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4]$$



It is customary to also **expand** $e(\rho, 0)$ and $S(\rho)$ around nuclear **saturation density**

$$\rho_0 \sim 0.16 \text{ fm}^{-3}$$

$$e(\rho, 0) = e(\rho_0, 0) + \frac{1}{2}K_0x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho - \rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}[\rho^3, \delta^4]$$

$K_0 \rightarrow$ how **compressible** is symmetric matter at ρ_0

J (a_A) $\rightarrow$ **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L (a_s) $\rightarrow$ **neutron pressure** in neutron matter at ρ_0

$$P(\rho = \rho_0, \delta = 0) = 0 \text{ MeV fm}^{-3}$$

Sum rules: m_1

Ground state gives access to excited state properties!!

→ **Sum Rules** or **moments** of the **strength function $S(E)$** :

$$S(E) = \sum_v |\langle v | \hat{F}_J | \tilde{0} \rangle|^2 \delta(E - E_v + E_0)$$

k-moment of $S(E)$:
$$m_k = \int dE E^k S(E) = \sum_v |\langle v | \hat{F}_J | \tilde{0} \rangle|^2 (E_v - E_0)^k$$

Example: Energy Weighted Sum Rule (EWSR)

$$m_1 = \sum_v (E_v - E_0) |\langle v | F | 0 \rangle|^2 = \langle 0 | F^\dagger [\mathcal{H}, F] | 0 \rangle$$

Written in terms of a **commutator** with the **Hamiltonian** evaluated in the **G.S. !!**

→ $[F, V] = 0$, if the excitation **operator** (F) **commutes** with the **interaction** (V) the **sum rule** will be **model independent!!**

$$\hat{F}_J^{(IS)} = \sum_{i=1}^A f_J(r_i) Y_{JM}(\hat{r}_i) \quad m_1^{(IS)}(J) = \frac{\hbar^2}{2m} \frac{A}{4\pi} (2J+1) \langle g_J \rangle,$$

$$\langle g_J \rangle = \frac{1}{A} \int g_J(r) \rho(r) d^3r,$$

$$g_J(r) = \left(\frac{df_J}{dr} \right)^2 + J(J+1) \left(\frac{f_J}{r} \right)^2$$

→ $[F, V] \neq 0$, if the excitation **operator does not commute** with the **interaction** the **sum rule** will be **model dependent** but still can be **used** to better **understand** nuclear **phenomenology**

$$\hat{F}_J^{(IV)} = \sum_{i=1}^A f_J(r_i) Y_{JM}(\hat{r}_i) \tau_z(i) \quad m_1^{(IV)}(J) = m_1^{(IS)}(J) (1 + \tilde{\kappa})$$

Model dependent term

Sum rules: m_{-1}

Ground state gives access to excited state properties!!

Theoretically, the total **photo-absorption cross section**, can be written as

$$\sigma_{\gamma-\text{abs}} = 4\pi^2\alpha \sum_{\nu} (E_{\nu} - E_0) |\langle \nu | F_{\text{dipole}} | 0 \rangle|^2$$

Dipole operator
subtract
CM motion

And, thus,

$$\alpha_D = 2 \sum_{\nu \neq 0} \frac{|\langle \nu | F_{\text{dipole}} | 0 \rangle|^2}{E_{\nu} - E_0} \equiv 2m_{-1}$$

$$\frac{eN}{A} \sum_{i=1}^Z r_i Y_{1M}(\hat{r}_i) - \frac{eZ}{A} \sum_{i=1}^N r_i Y_{1M}(\hat{r}_i)$$

Considering the G.S. perturbed by an **external field** λF (with $\lambda \rightarrow 0$). $\langle H \rangle = \langle H_0 + \lambda F_{\text{dipole}} \rangle$ can be written as:

$$\delta \langle \mathcal{H} \rangle = \langle \mathcal{H} \rangle - \langle \mathcal{H}_0 \rangle = \lambda^2 \sum_{\nu \neq 0} \frac{|\langle \nu | F | 0 \rangle|^2}{E_{\nu} - E_0} + \mathcal{O}(\lambda^3) = \lambda^2 m_{-1} + \mathcal{O}(\lambda^3)$$

$$m_{-1} = \frac{1}{2} \frac{\partial^2 \langle \mathcal{H} \rangle}{\partial \lambda^2} \Big|_{\lambda=0}$$

**Dielectric
Theorem**

Dielectric theorem applied to the Droplet model Hamiltonian

→ Calculate the polarizability (α), **proportional to m_{-1}** from the **dielectric theorem** and **Droplet Model**

$$m_{-1} \approx \frac{A \langle r^2 \rangle^{1/2}}{48J} \left(1 + \frac{15}{4} \frac{J}{Q} A^{-1/3} \right)$$

J. Meyer, P. Quentin, and B. Jennings, [Nucl. Phys. A 385, 269](#)

$$a_{\text{sym}}(A) = \frac{J}{1 + x_A}, \quad \text{with} \quad x_A = \frac{9J}{4Q} A^{-1/3}, \quad \Delta r_{np}^{\text{DM}} = \frac{2r_0}{3J} [J - a_{\text{sym}}(A)] A^{1/3} (I - I_C)$$

$$\alpha_D \approx \frac{A \langle r^2 \rangle}{12J} \left[1 + \frac{5}{2} \frac{\Delta r_{np} + \sqrt{\frac{3}{5}} \frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2 \rangle^{1/2} (I - I_C)} \right]$$

Polarizability must increase with the mass (for the dipole $A^{5/3}$, for the quadrupole $A^{7/3}$ and so on) and **surface symmetry energy** (J/Q) and **decrease** with the **bulk symmetry energy** (J)

Density Functional Theory: a microscopic theory

DENSITY FUNCTIONAL THEORY

Hohenberg-Kohn theorems

P.Hohenberg, W. Kohn, Phys. Rev. 136, B864 (1964)

→ Assuming a system of **interacting fermions** in a confining **external potential**, there exist a **universal** functional **$F[\rho]$** of the fermion density **ρ** :

$$E[\rho] = \langle \Psi | T + V + V_{\text{ext}} | \Psi \rangle = F[\rho] + \int V_{\text{ext}}(r) \rho(r) d\vec{r}$$

→ and it can be shown that

$$\min_{\Psi} \langle \Psi | T + V + V_{\text{ext}} | \Psi \rangle = \min_{\rho} E[\rho]$$

so **$E[\rho]$** has a **minimum** for the **exact ground-state density** where it assumes the **exact energy** as a value.

Kohn-Sham realization

$$F[\rho] \rightarrow T_{\text{non-int.}}[\rho] + V_{\text{KS}}[\rho]$$

In nuclei no need of external confining potential

For **any interacting system**, there exists a **local single-particle potential** $V_{\text{KS}}(\mathbf{r})$, such that the **exact ground-state density** of the **interacting system** equals the **ground-state density** of the **auxiliary non-interacting system**:

$$\rho_{\text{exact}}(\vec{r}) = \rho_{\text{KS}}(\vec{r}) = \sum_{i=1}^A |\phi(\vec{r})|^2$$

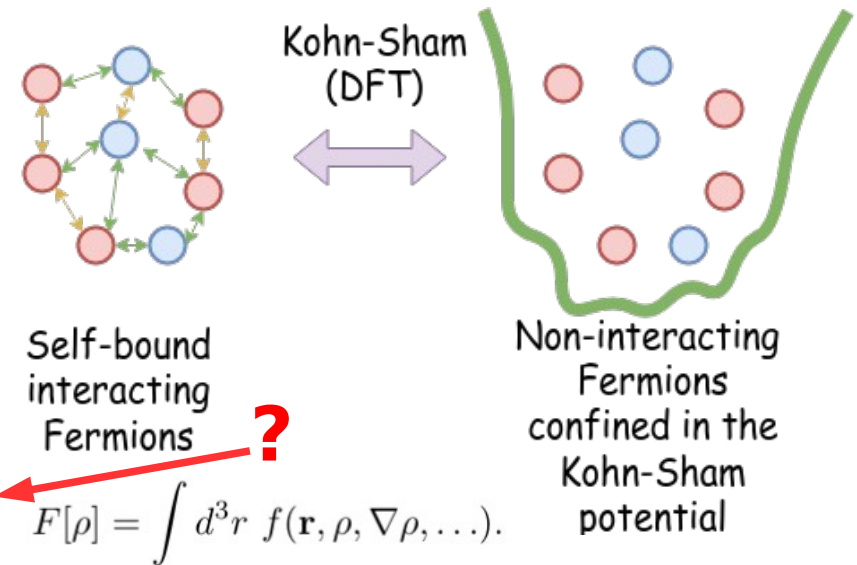
where ϕ are single-particle orbitals and the total wave-function correspond to a Slater determinant. The **$E[\rho]$ is unique**

$$E[\rho] = T[\rho] + \int V_{\text{KS}}(\vec{r}) \rho(\vec{r}) d\vec{r}$$

where **$T[\rho]$ is the kinetic energy of the non-interacting system** and for which the variational equation

$$0 = \frac{\delta E[\rho]}{\delta \rho} = \frac{\delta T[\rho]}{\delta \rho} + V_{\text{KS}}$$

yields to the **exact ground state density and energy**



Time dependent DFT for the study of GR

Linear Response Theory (Ring&Schuck)

Perturbing the initial static Hamiltonian H_0 with a small **time dependent operator $F(t)$** :

$$\mathcal{H} = \mathcal{H}_0 + F(t) \quad F(t) = f \exp(-i\omega t) + f^\dagger \exp(i\omega t)$$

Will produce variations on the **static density ρ_0** linear with the **external operator $F(t)$** in first approximation:

$$\delta\rho(t) = \delta\rho \exp(-i\omega t) + \delta\rho^\dagger \exp(i\omega t)$$

Writing the **Schroedinger equation using commutators**

$$\mathcal{H} = \sum_i h(i) \quad h\Psi(t) = i\hbar\dot{\Psi}(t) \rightarrow [h, \rho] = i\hbar\dot{\rho}$$
$$h_0\Psi = \varepsilon\Psi \rightarrow [h_0, \rho_0] = 0 \quad [h_0 + F(t), \rho_0 + \delta\rho(t)] = i\hbar\delta\dot{\rho}$$

Time dependent DFT for the study of GR

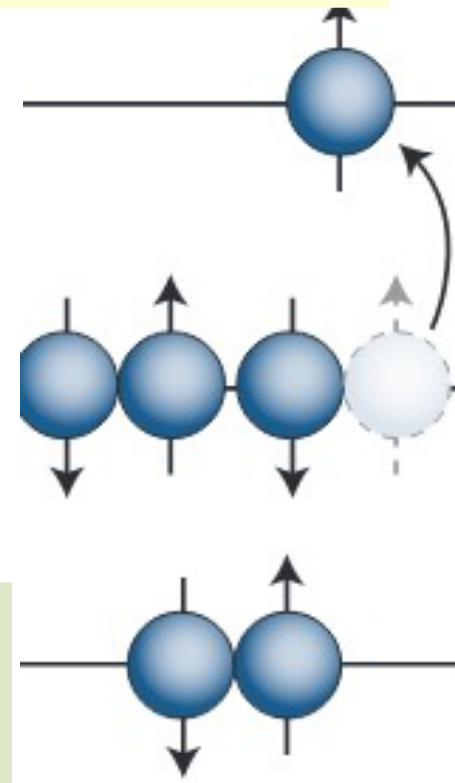
Linear Response Theory (Ring&Schuck)

Keeping the **linear terms** in the **perturbation** (F) and imposing that a Slater determinant (only for **particle-hole** or **hole-particle** excitations [**GR**→ **Many coherent ph excitations!**]) one could find:

$$(\omega - \epsilon_m + \epsilon_i)\delta\rho_{mi} = f_{mi} + \sum_{m'i'} V_{mi'im'}\delta\rho_{m'i'} + V_{mm'ii'}\delta\rho_{im'}$$

$$(\omega - \epsilon_i + \epsilon_m)\delta\rho_{im} = f_{im} + \sum_{m'i'} V_{imi'mm'}\delta\rho_{m'i'} + V_{im'mi'}\delta\rho_{im'}$$

$$V_{kl'l'k'} = \sum_{kk'} \left. \frac{\partial h_{kl}}{\partial \rho_{k'l'}} \right|_{\rho^{(0)}} = \frac{\partial^2 E}{\partial \rho_{lk} \partial \rho_{k'l'}} \bigg|_{\rho^{(0)}}$$



For **F**→ **0** and solving the Eqs. for **δp** one finds the **Random Phase Approximation** where the knowledge of **E[ρ]** is **sufficient**, no need to impose H.

Some selected results

Electric Dipole Polarizability: Do we understand it from theory?

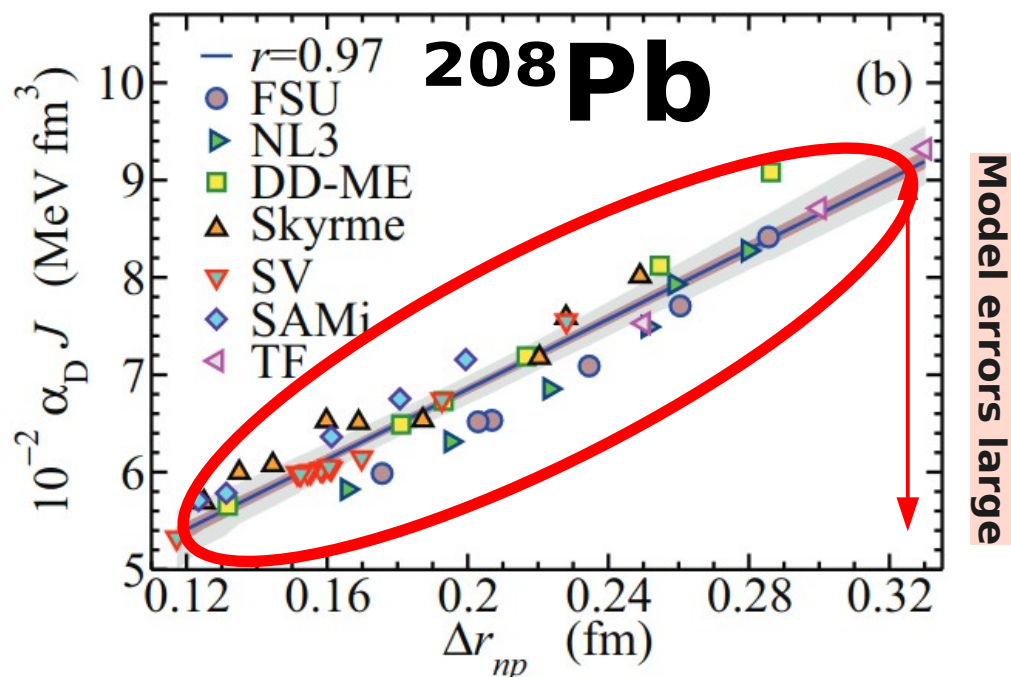
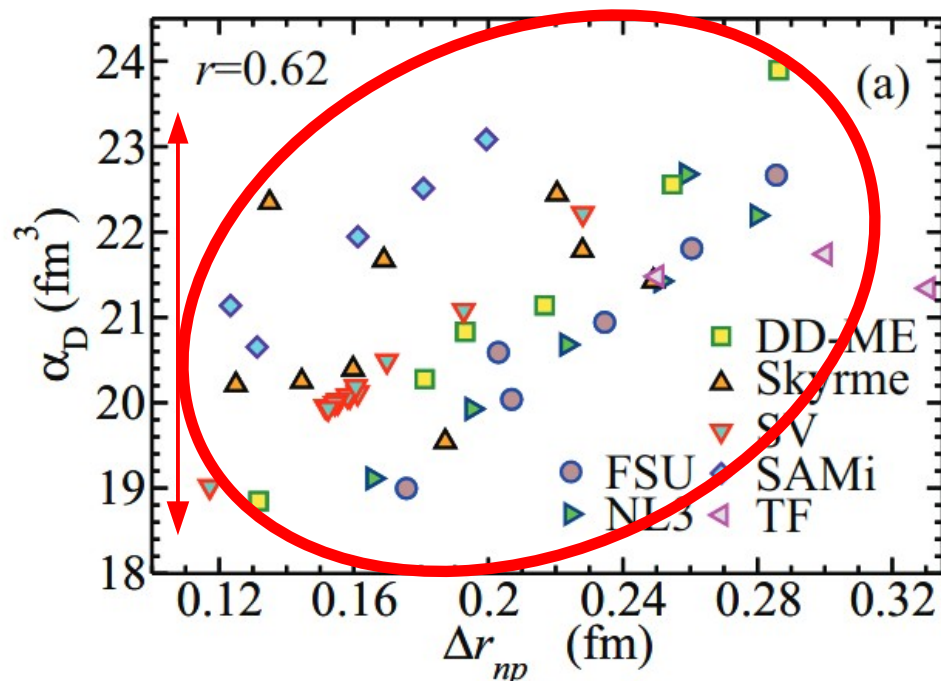
Applying the **dielectric theorem** to the **Droplet Model** Hamiltonian (first Migdal and latter on Meyer et al. NPA385, 269) one can find (shown before)

$$\alpha_D \approx \frac{A \langle r^2 \rangle}{12J} \left[1 + \frac{5 \Delta r_{np} + \sqrt{\frac{3}{5} \frac{e^2 Z}{70J}} - \Delta r_{np}^{\text{surface}}}{\langle r^2 \rangle^{1/2} (I - I_C)} \right]$$

$J = e(\text{PNM}) - e(\text{SNM}) \rightarrow$
Symmetry energy at ρ_0
 $\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2} \rightarrow$
Neutron skin thickness

Using microscopic calculations

(Energy Density Functionals $\leftrightarrow$ Hartree-Fock+Random Phase Approximation)



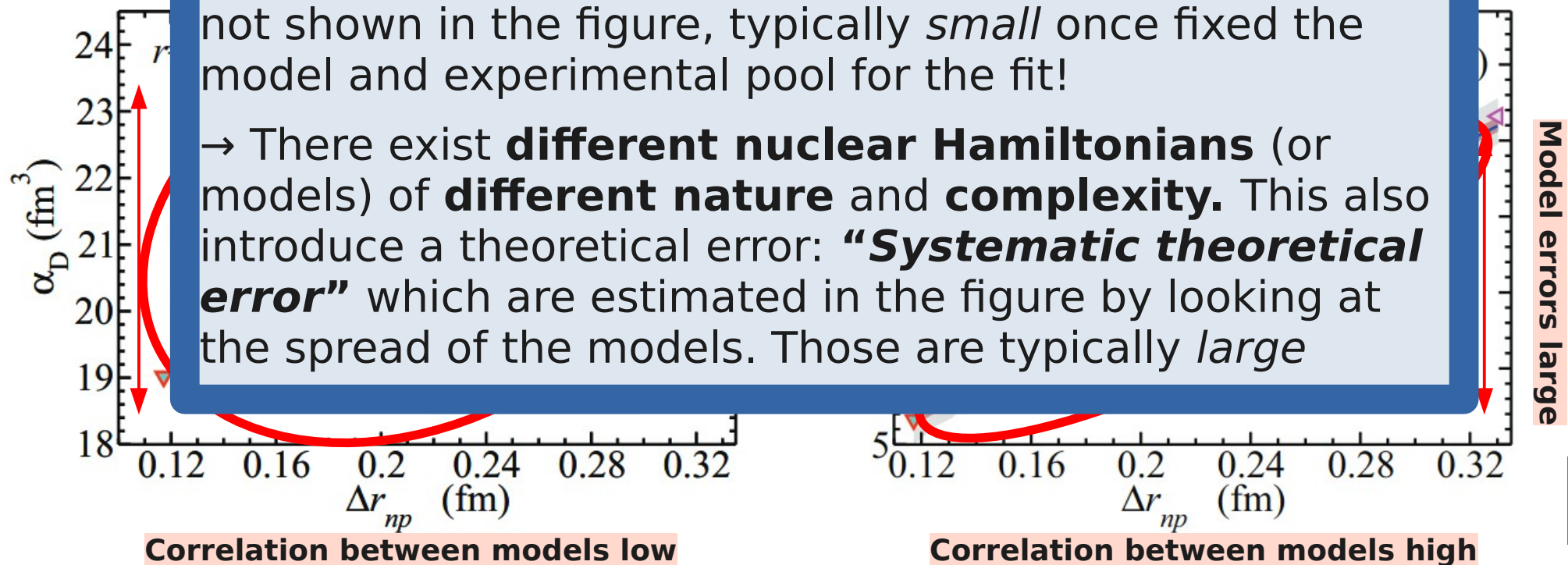
Electric Dipole Polarizability: Do we understand it from theory?

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WARNING: one must remember that (nowadays) **all nuclear models are effective models.**

→ Parameters of the model are not fundamental constants but are **fitted** to reproduce some experimental data → “**Statistical theoretical**” errors not shown in the figure, typically *small* once fixed the model and experimental pool for the fit!

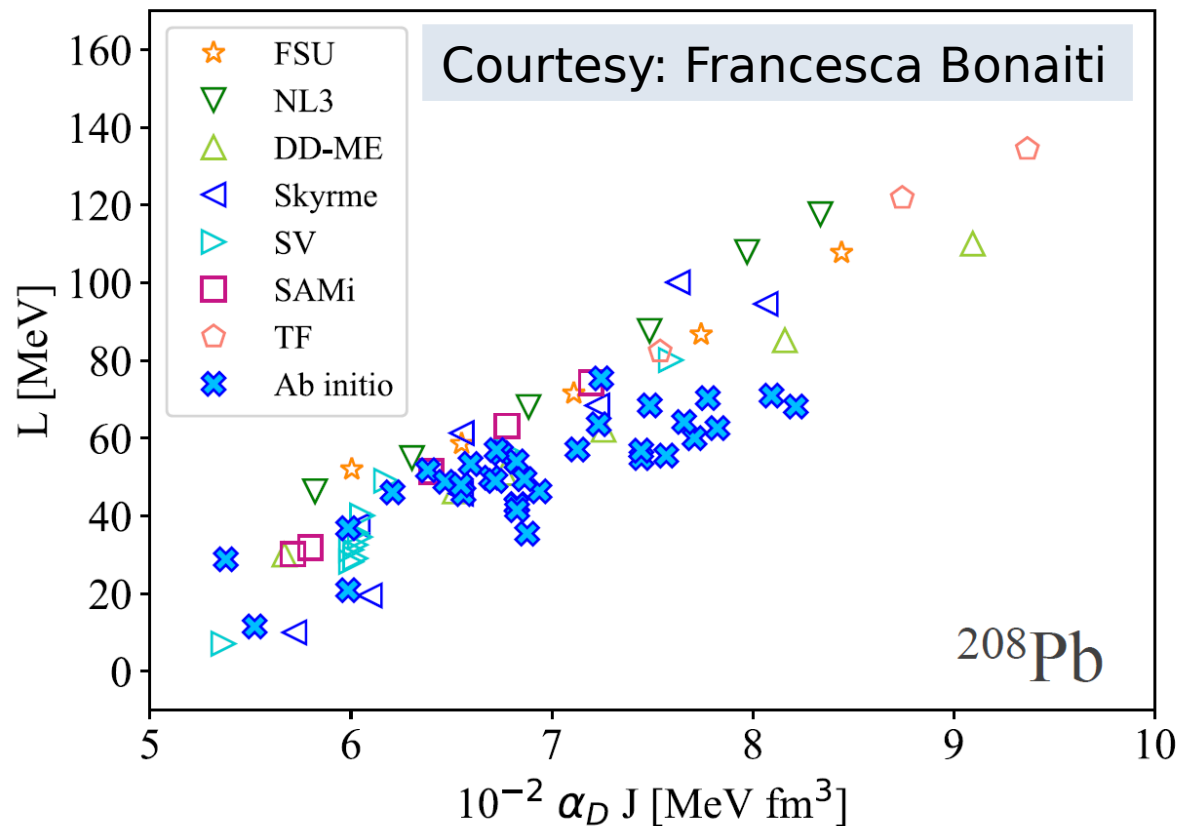
→ There exist **different nuclear Hamiltonians** (or models) of **different nature** and **complexity**. This also introduce a theoretical error: “**Systematic theoretical error**” which are estimated in the figure by looking at the spread of the models. Those are typically *large*



EDFs are consistent with Droplet Model

What about “ab initio” models?

What happens if we add to the previous results based on **DFT** from **χ -EFT**?



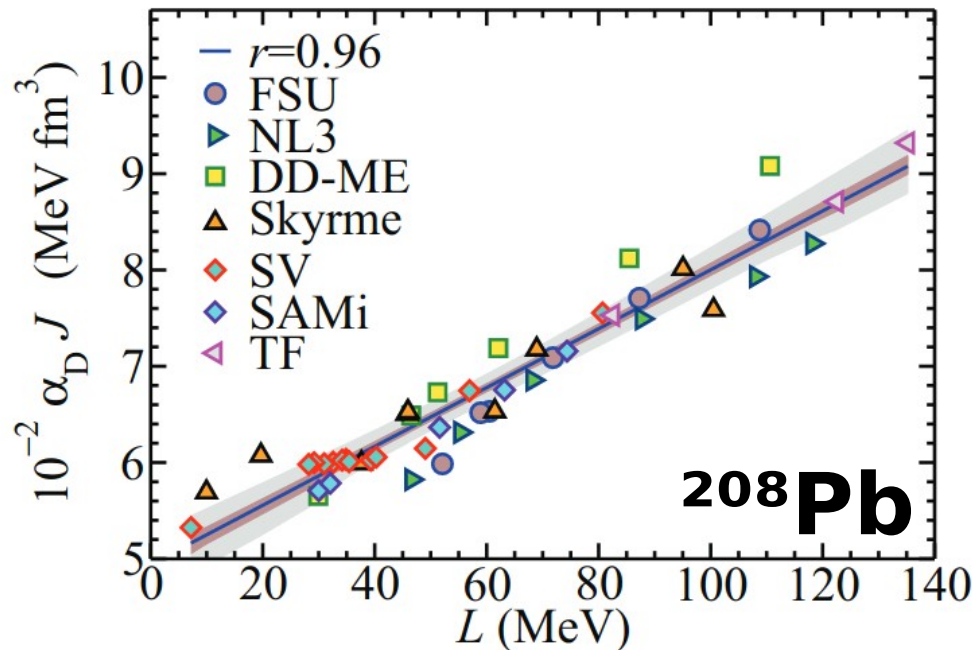
DFT data from:
Roca-Maza, Phys. Rev. C **88**,
024316 (2013)

Ab initio data from:
Hu, Nat. Phys. **18**, 1196-1200
(2022)

It looks consistent, may be we understood something

Now we are allowed to study the consequences of what we have learned on the EoS

Determination of the J vs L relation from experimental data according to EDFs



$$\begin{aligned}
 J &= 25.0(2) + 0.19(2)L && \text{for } ^{68}\text{Ni}, \\
 J &= 25.4(1.1) + 0.17(1)L && \text{for } ^{120}\text{Sn}, \\
 J &= 24.5(8) + 0.17(1)L && \text{for } ^{208}\text{Pb}.
 \end{aligned}$$

Assuming Taylor expansion around ρ_0 :

$$S(\langle\rho\rangle) \approx J - L \frac{\rho_0 - \langle\rho\rangle}{3\rho_0} \rightarrow J \approx S(\langle\rho\rangle) + \frac{\rho_0 - \langle\rho\rangle}{3\rho_0} L$$

one can qualitatively understand the result!!

$$S(\langle\rho\rangle \approx 0.08 \text{ fm}^{-3}) \approx 25 \text{ MeV}$$

Current applications to the measurement or calculation of the nuclear polarizability

Polarizability is sensitive to the **neutron skin thickness** and **neutron distribution**. If accurately determined we can learn about **isospin symmetry breaking** effects in the nuclear medium $\leftrightarrow$ **restoration of chiral symmetry?** And on the **weak charge of the nucleus** $\leftrightarrow$ **weak mixing angle?**

Effects on **atomic energy levels** due to nuclear polarization (electronic or muonic atoms) $\leftrightarrow$ help in accurate **determination** of electric **charge radii?**

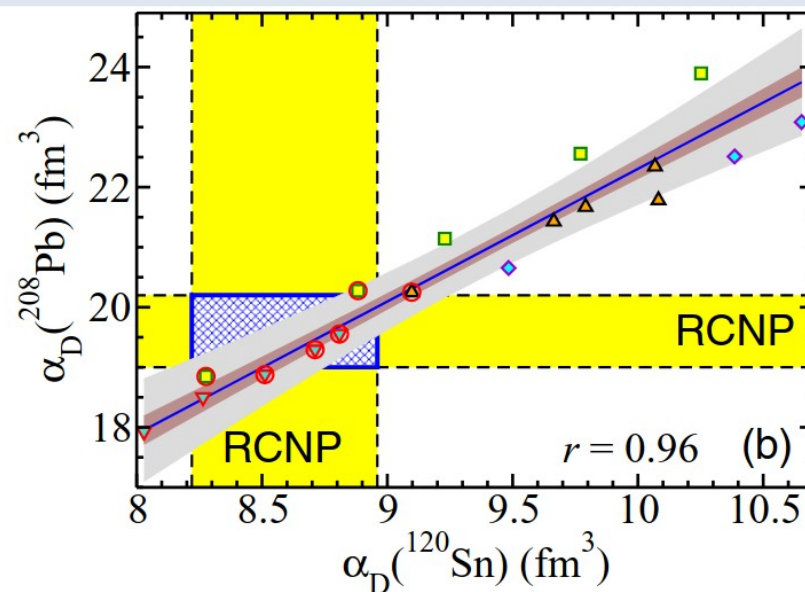
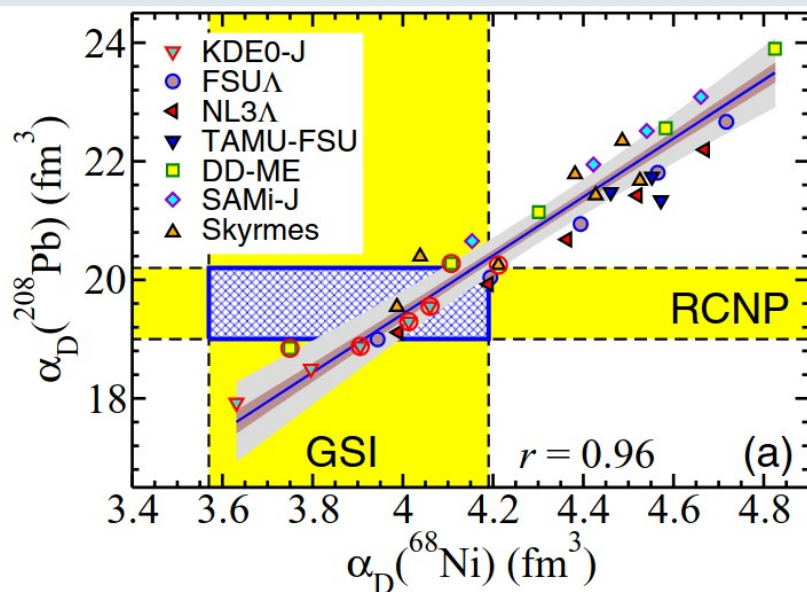
Learning about **nuclear polarizability** could help understanding the **tidal deformability** (=quadrupole polarizability) in neutron stars

Collaborators

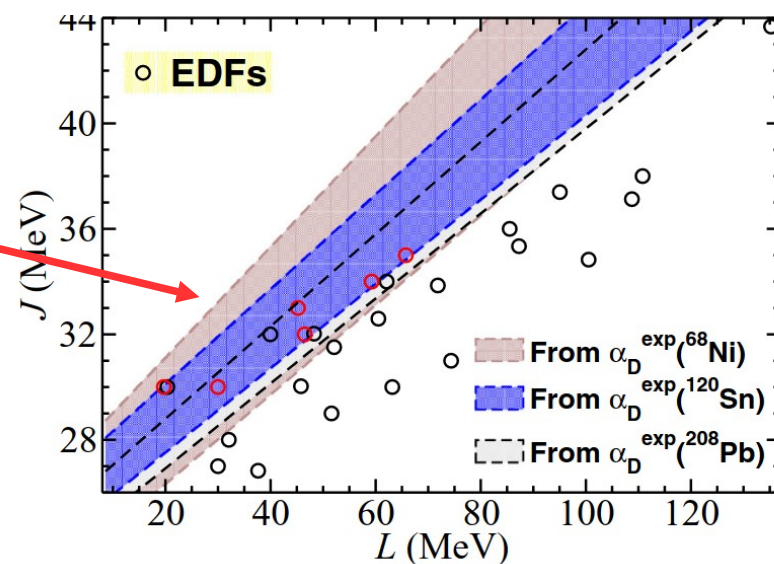
- Gianluca **Colò** (University of Milan)
- Hiroyuki **Sagawa** (University of Aizu & RIKEN)
- Tomoya **Naito** (University of Tokyo & RIKEN)
- Shihang **Shen** (Forschungszentrum Jülich)
- Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)
- Jorge **Piekarewicz** (Florida State University)
- Nils **Paar** & Dario **Vretenar** (University of Zagreb)
- Bijay K. **Agrawal** (Saha Institute of Nuclear Physics)
- P.-G. **Reinhard** (University of Erlangen-Nürnberg)
- Witold **Nazarewicz** (FRIB and Michigan State University)

Are these results robust?

Selection of EDFs (red circles) compatible with experimental data

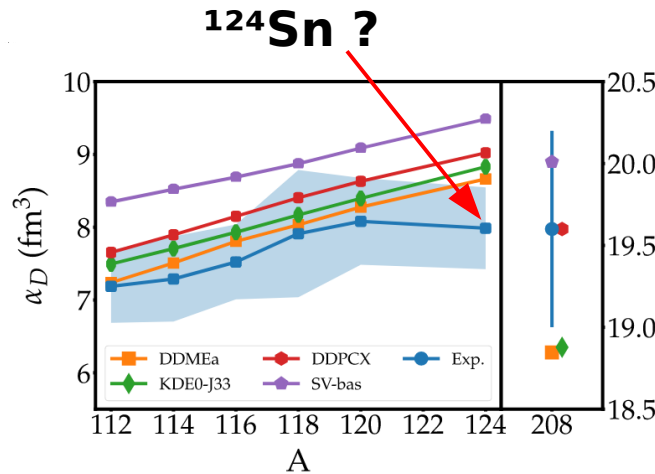
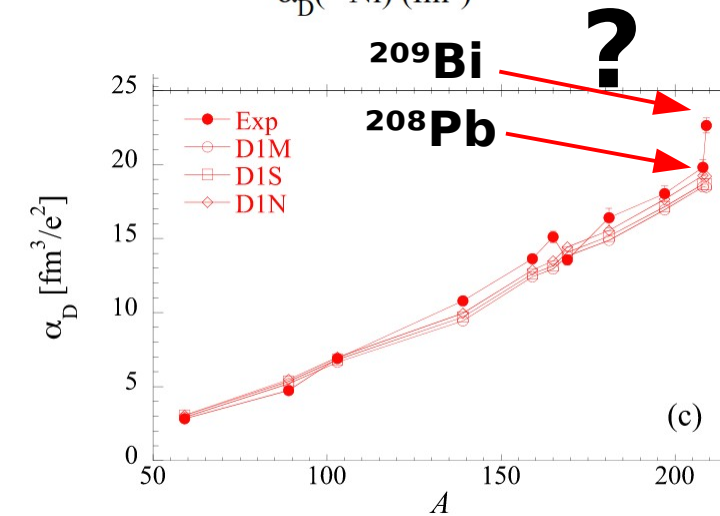
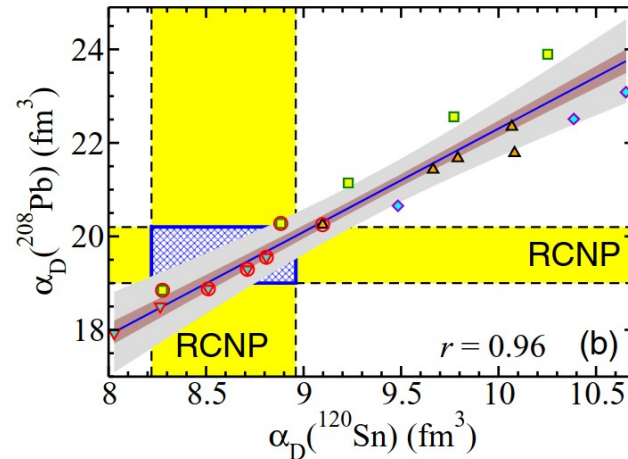
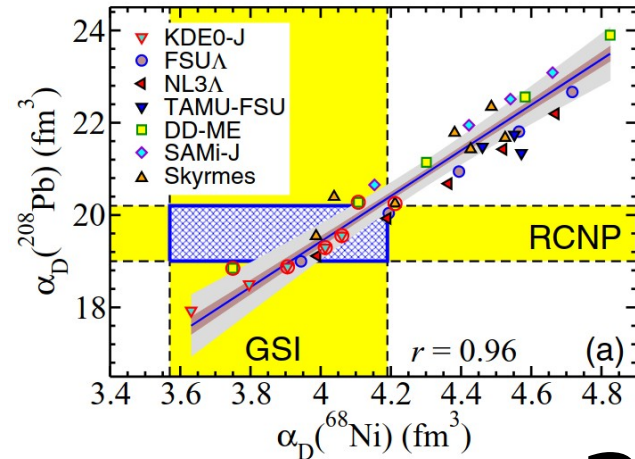


Warning: Some EDFs not compatible with experiment (black circles) are within bands, but most of them outside!



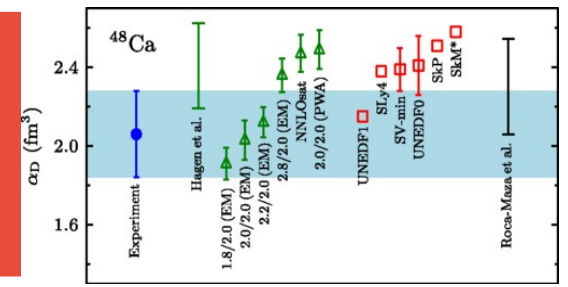
Electric Dipole Polarizability: but do we really really understand it?

X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

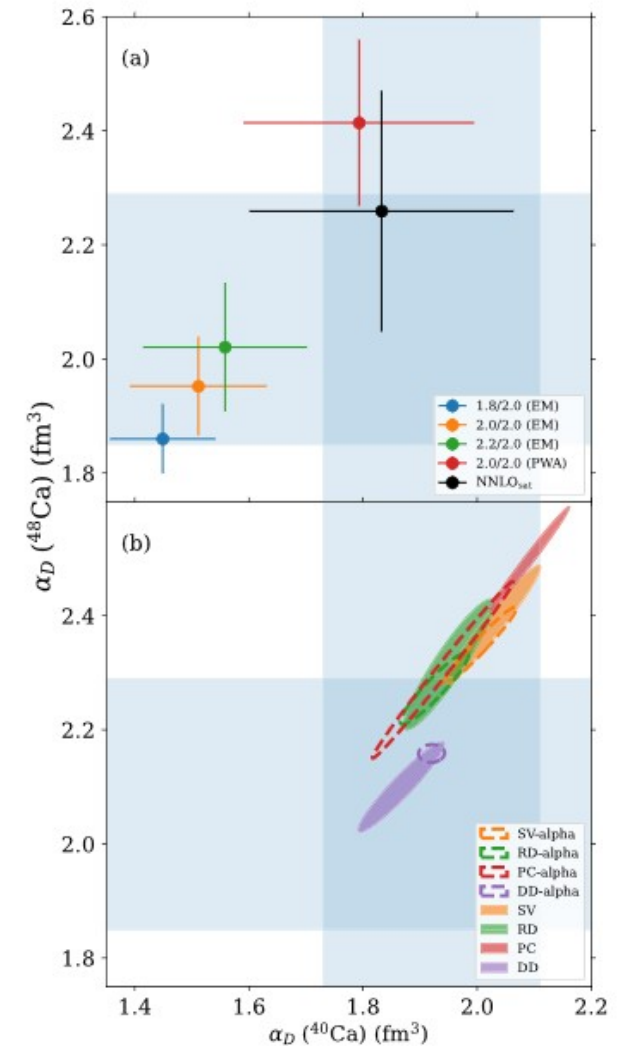


S. Bassauer, P. von Neumann-Cosel, P.-G. Reinhard et al.
Physics Letters B **810** (2020) 135804

S. Goriely, S. Péru, G. Colò, X. Roca-Maza, I. Gheorghe, D. Filipescu, and H. Utsunomiya
Phys. Rev. C **102**, 064309 – Published 9 December 2020

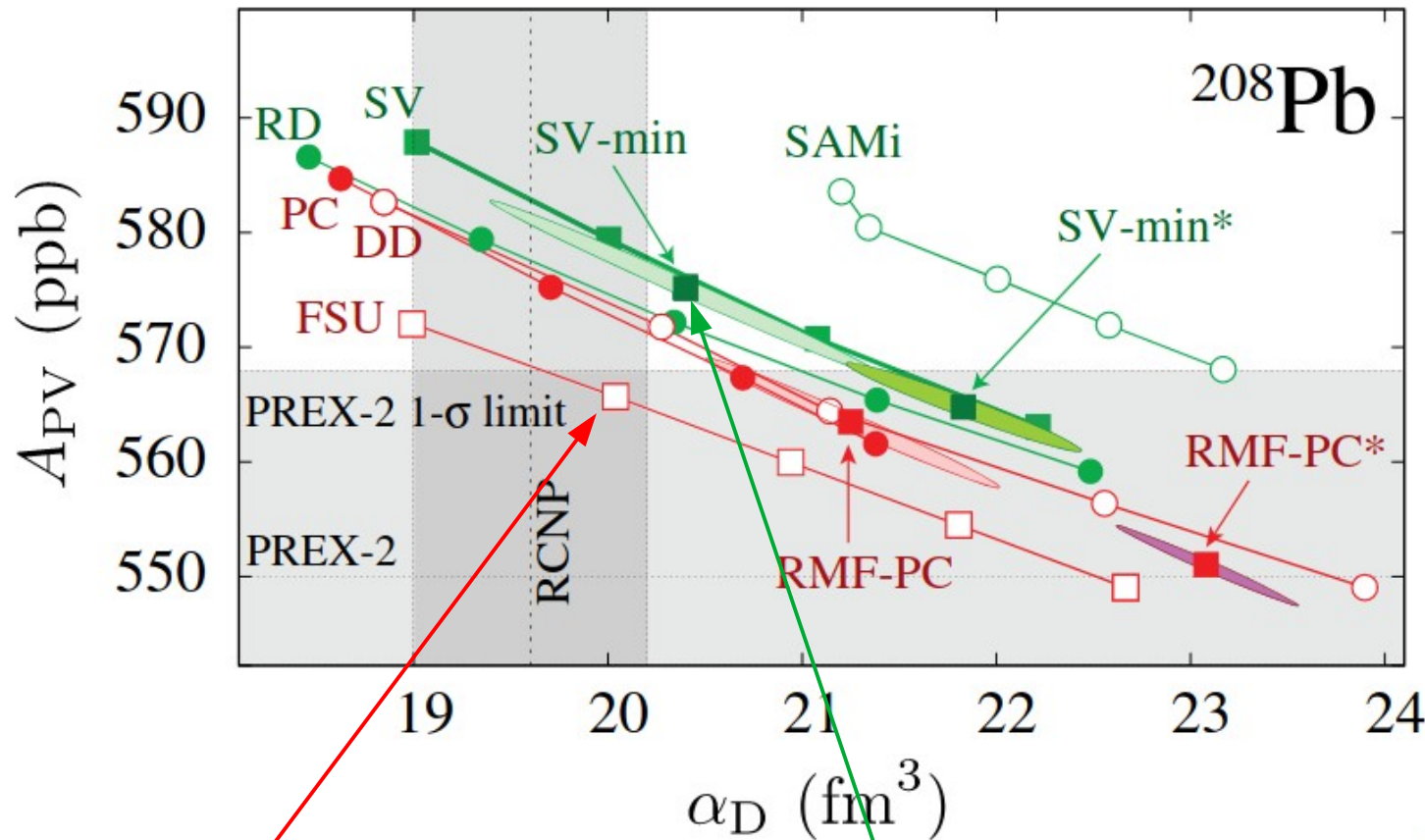


J. Birkhan, M. Miorelli, S. Bacca, S. Bassauer, C. A. Bertulani, G. Hagen, H. Matsubara, P. von Neumann-Cosel, T. Papenbrock, N. Pietralla, V. Yu. Ponomarev, A. Richter, A. Schwenk, and A. Tamii
Phys. Rev. Lett. **118**, 252501 – Published 23 June 2017



R. W. Fearick, P. von Neumann-Cosel, S. Bacca, J. Birkhan, F. Bonaiti, I. Brandherm, G. Hagen, H. Matsubara, W. Nazarewicz, N. Pietralla, V. Yu. Ponomarev, P.-G. Reinhard, X. Roca-Maza, A. Richter, A. Schwenk, J. Simonis, and A. Tamii
Phys. Rev. Research **5**, L022044 – Published 31 May 2023

And if we combine A_{PV} and α_D in ^{208}Pb ?



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. 127, 232501 (2021)

- Models with an * contain A_{pv} in ^{208}Pb in the fitting protocol
- Models connected by lines: family
- SV, PC and RD models fitted to the same data

SV-min:

$$r_{\text{skin}} = 0.19 \pm 0.02 \text{ fm}$$

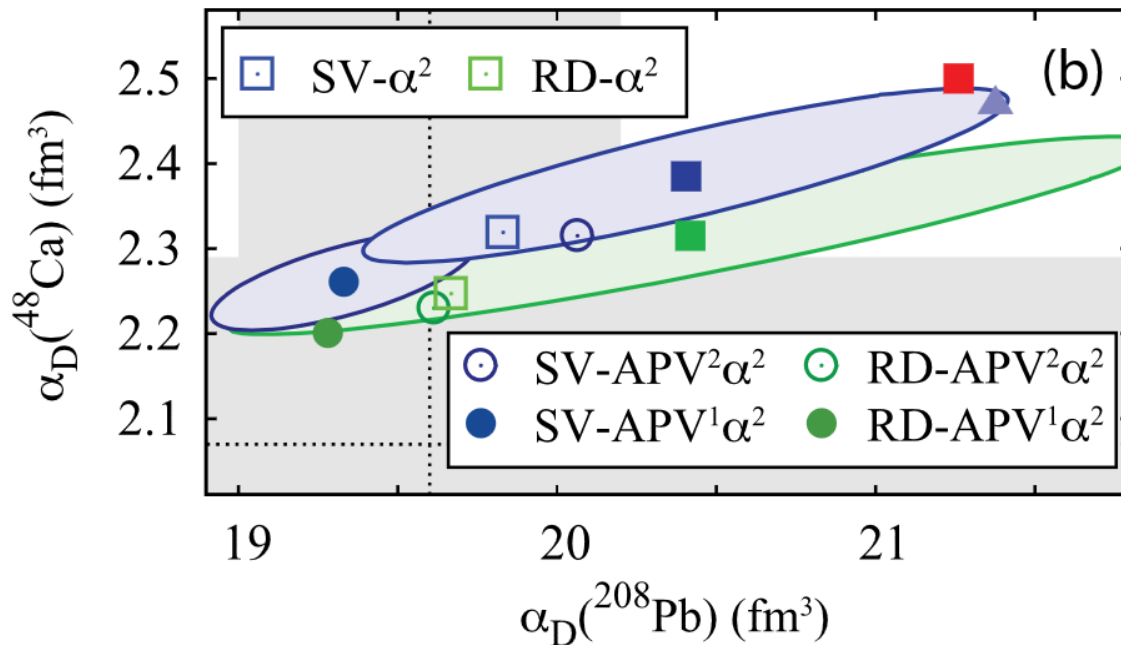
$$L = 54 \pm 8 \text{ MeV}$$

B and **Rch** away from “expected” EDF accuracy for B and Rch



Theoretical and experimental 1σ errors overlap for SV-min
But not overlap both exp.

Summary: model performance A_{PV} (sensitive to Δr_{np}) and α_D (sensitive to J and Δr_{np}) in ^{48}Ca and ^{208}Pb



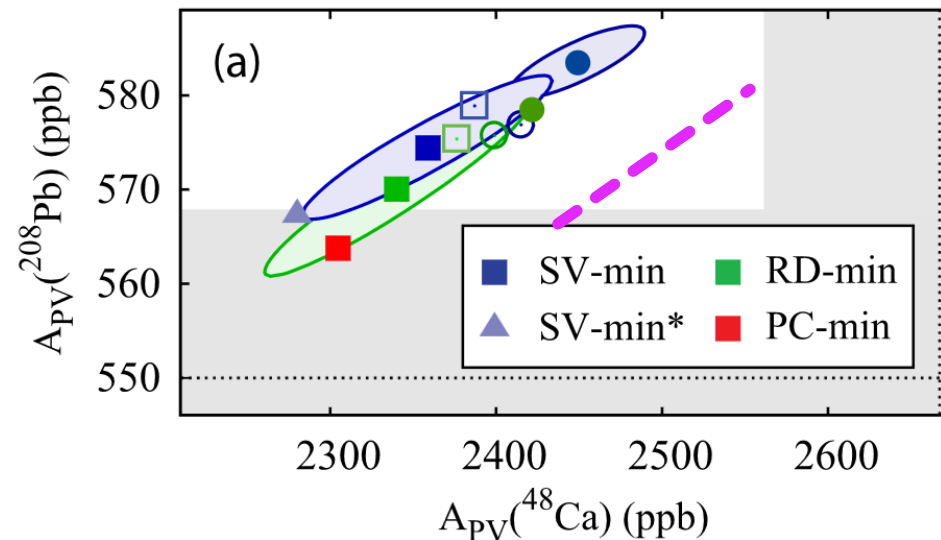
No simultaneous description of **parity violating asymmetries** (ground state observable) → point to a **deficient understanding** of **neutron skins**

Simultaneous description of **dipole polarizabilities** → point to a **good understanding** of **symmetry energy (J)** and **neutron skins (Δr_{np})**

Ab-initio (B. Hu) Nature Physics (2022)

$$\alpha_D(^{48}\text{Ca}) \quad 2.30^{+0.31}_{-0.26}$$

$$\alpha_D(^{208}\text{Pb}) \quad 22.6^{+2.1}_{-1.8}$$



Magenta dashed lines from extrapolated Δr_{np} given in G. Hagen et al. Nature Physics 12, 186-190 (2016) and H. Bu et al. Nature Physics (2022)