Parity violating asymmetry and dipole polarizabily: how do they reconcile with each other?

Xavier Roca-Maza

VIIth Topical Workshops on Modern Aspects of Nuclear Structure

Bormio, February 3rd - 8th

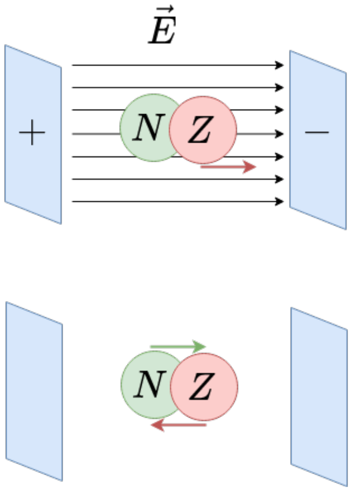
Electric Dipole Polarizability: introduction

The **electric dipole polarizability** measures the **tendency** of the nuclear **charge distribution** to be **distorted**

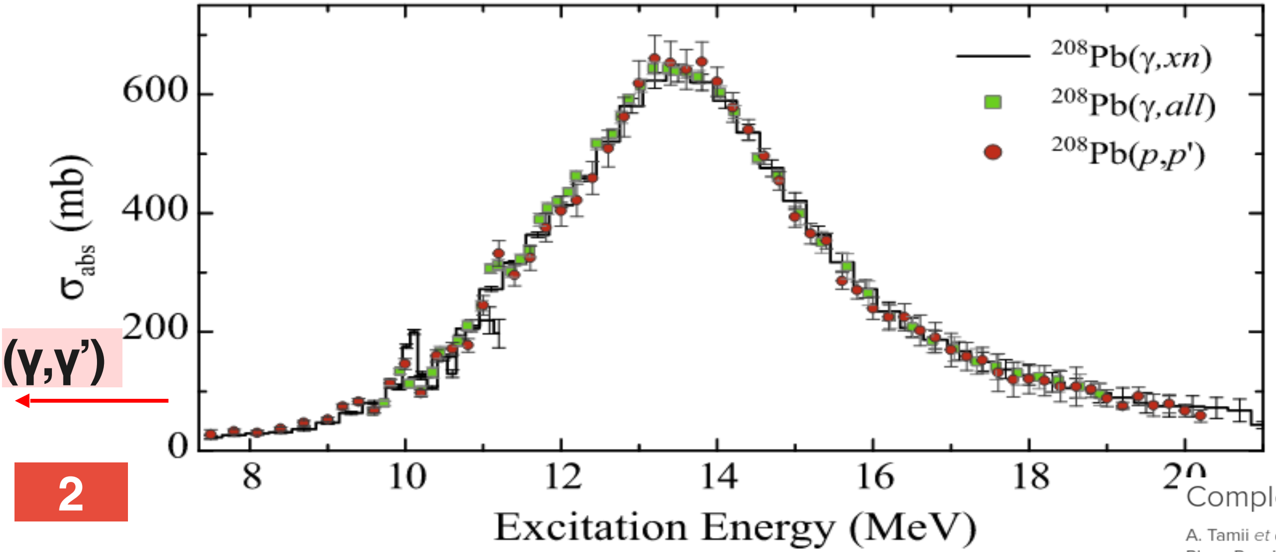
$$\alpha = \frac{\text{electric dipole moment}}{\text{external electric field applied}}$$

Microscopically, it relates with the **photo-absorption cross-section**

$$\alpha_D = \frac{\hbar c}{2\pi^2 e^2} \int \frac{\sigma_{\text{abs}}}{\omega^2} d\omega,$$



Measured using polarized proton scattering at very forward angles (dominated by E1 and M1 well separated)



$$\boxed{\alpha_D(^{208}\text{Pb}) = 19.6 \pm 0.6 \text{ fm}^3}$$

Complete Electric Dipole Response and the Neutron Skin in ^{208}Pb
A. Tamii *et al.*
Phys. Rev. Lett. **107**, 062502 – Published 3 August 2011

Electric Dipole Polarizability: simple model & correlations

Applying the **dielectric theorem** to the **Droplet Model** Hamiltonian (first Migdal and latter on Meyer et al. NPA385, 269) one can find

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1+\frac{5}{2}\frac{\Delta r_{np}+\sqrt{\frac{3}{5}\frac{e^2 Z}{70J}}-\Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I-I_C)}\right]$$

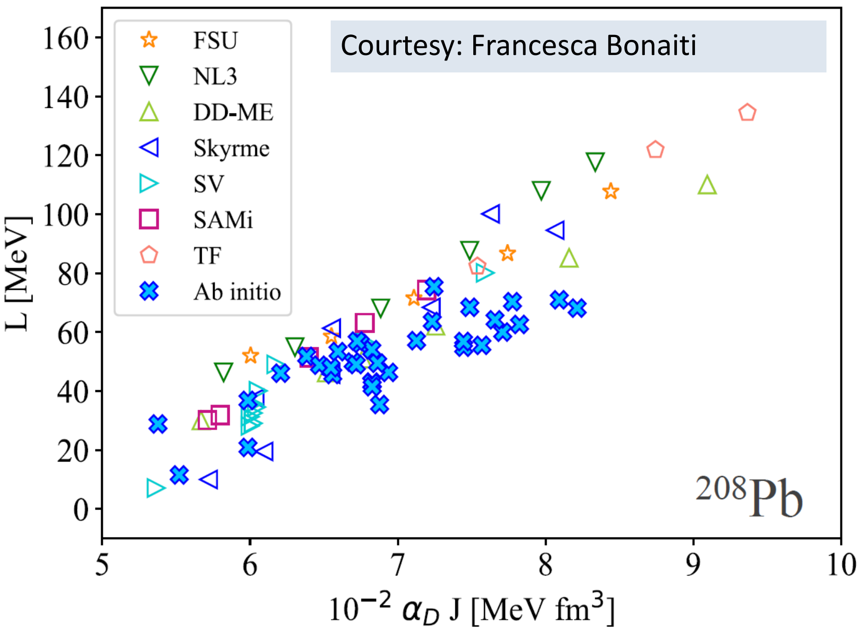
J = e(PNM)–e(SNM) → Symmetry energy at ρ_0

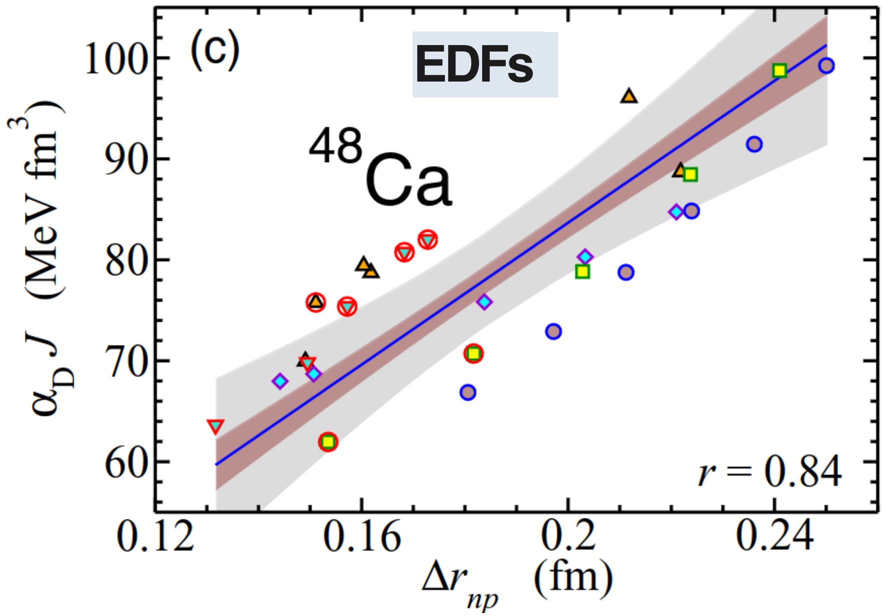
$\Delta r_{np} \equiv \langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$ → Neutron skin thickness

Using microscopic calculations (Energy Density Functionals & Ab initio)

DFT data from: Roca-Maza, Phys. Rev. C **88**, 024316 (2013)

Ab initio data from: Hu, Nat. Phys. **18**, 1196-1200 (2022)





Phys. Rev. C **92**, 064304 – Published 8 December 2015

Parity Violating Asymmetry: introduction

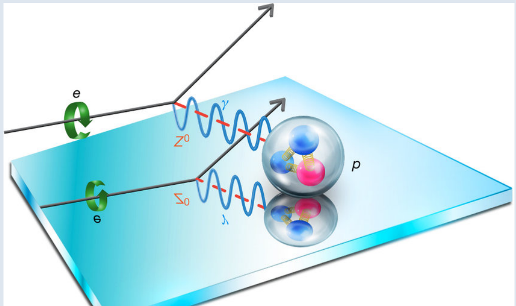
Elastic electron scattering by nuclei:

→ **Parity conserving**

Exchange of γ that essentially couples to protons $Q_p^c=1$; $Q_n^c=0$

→ **Parity violating**

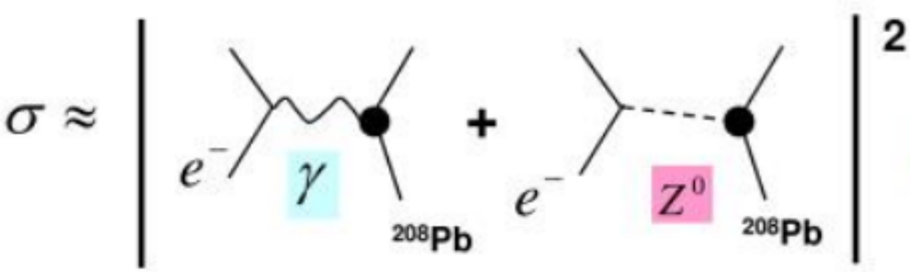
Exchange Z_0 that essentially couples to neutrons $Q_p^W=0.07$; $Q_n^W=-0.99$



Nature **557**, 207–211 (2018)

Ultra-relativistic electrons ($m_e \to 0$) with spin **aligned** (+) or **anti-aligned** (-) to the beam line, in approaching the nucleus:

$$\left[\vec{\alpha}\cdot\vec{p} + V_{\mathrm{Coulomb}}(\vec{r}) \pm V_{\mathrm{Weak}}(\vec{r})\right]\Psi_{\pm} = E\Psi_{\pm}$$



One can get **advantage** form this **interference pattern** between **electromag.** and **weak** interact.

$$A_{PV} = \frac{\frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega}}{\frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega}} \approx \frac{\text{Weak}}{\text{Coulomb}}$$

Parity Violating Asymmetry: theory

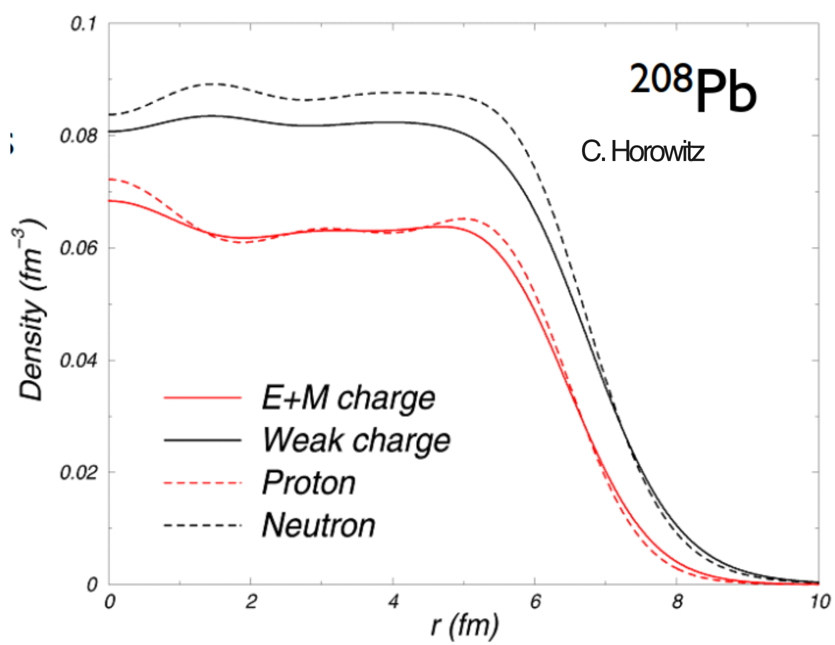
To solve the **Dirac equation**

$$\left[\vec{\alpha}\cdot\vec{p} + V_{\text{Coulomb}}(\vec{r}) \pm V_{\text{Weak}}(\vec{r})\right]\Psi_{\pm} = E\Psi_{\pm}$$

The main inputs are the **electromagnetic ρ_{ch}** and **weak charge ρ_{W}** distributions

$$V_{\text{Coulomb}}(\vec{r}) = Z\alpha\int\frac{\rho_{\text{ch}}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d\vec{r}\,' \qquad V_{\text{Weak}}(\vec{r}) = \frac{G_F}{2\sqrt{2}}\rho_{\text{W}}(\vec{r})$$

We need to know the **neutron and proton distributions** in nuclei and the **electromagnetic and weak** charge **form factors** of the **neutron** and the **proton**



Parity Violating Asimmetry: simple model & correlations

The **parity violating asymmetry** within the **PWBA:**

$$A_{PV}^{\mathrm{PWBA}} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[\underbrace{4\sin^2\theta_W}_{\approx 1} + \frac{F_n(Q)-F_p(Q)}{F_p(Q)}\right]$$

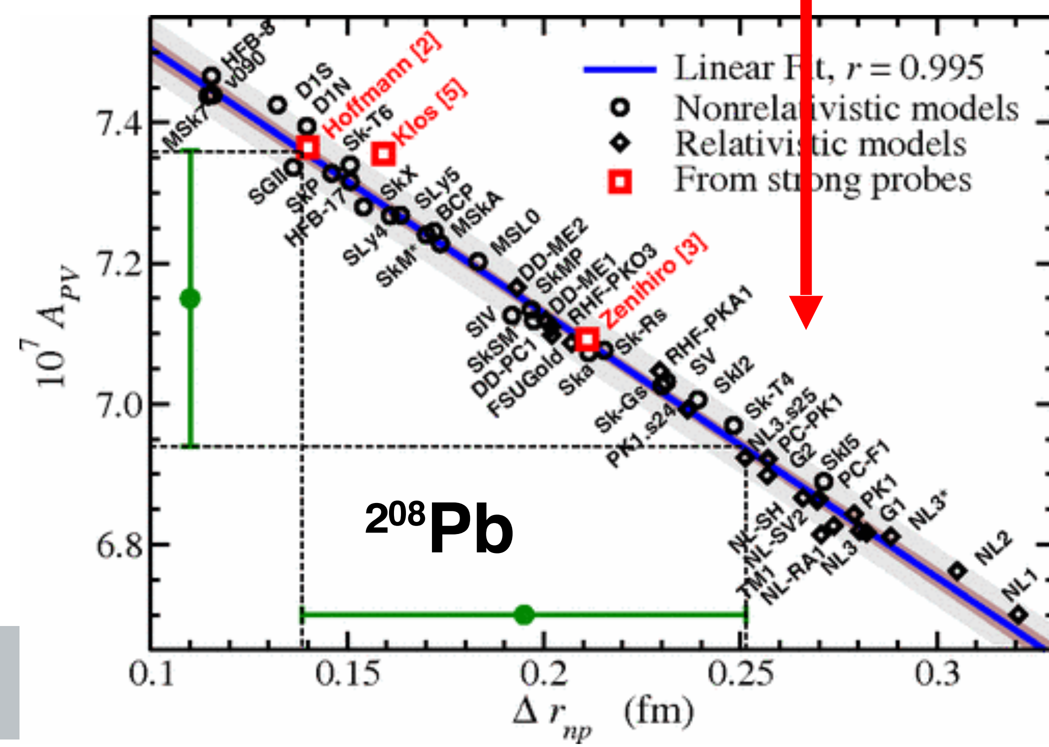
$$\xrightarrow{Q\to 0} \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[1 - \frac{Q^2\langle r_p^2\rangle}{3}\frac{\Delta r_{np}}{\langle r_p^2\rangle^{1/2}}\right]$$

30% due to Coulomb distortions (PREX) does not blur **the main physics**

The **parity violating asymmetry** within the **DWBA:**

○ ◇ → Energy Density Functionals

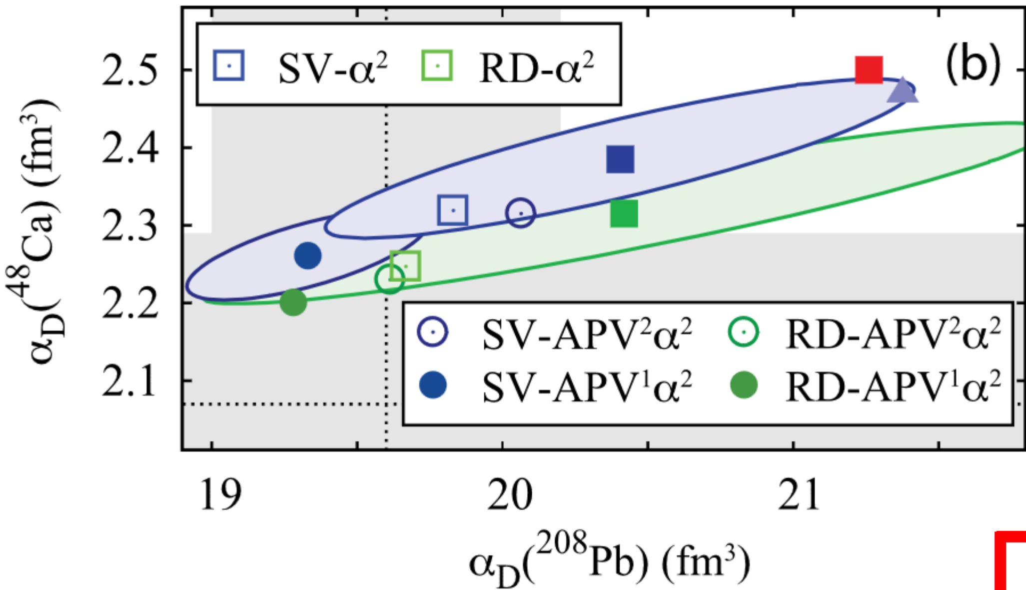
□ → Expeirmental ρ_n form hadronic probes + ρ_p from e⁻ scattering



Neutron Skin of ^{208}Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment

X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda
Phys. Rev. Lett. **106**, 252501 – Published 21 June 2011

Summary: model performance
A_{PV} (sensitive to Δr_{np}) and α_D (sensitive to J and Δr_{np}) in ^{48}Ca and ^{208}Pb



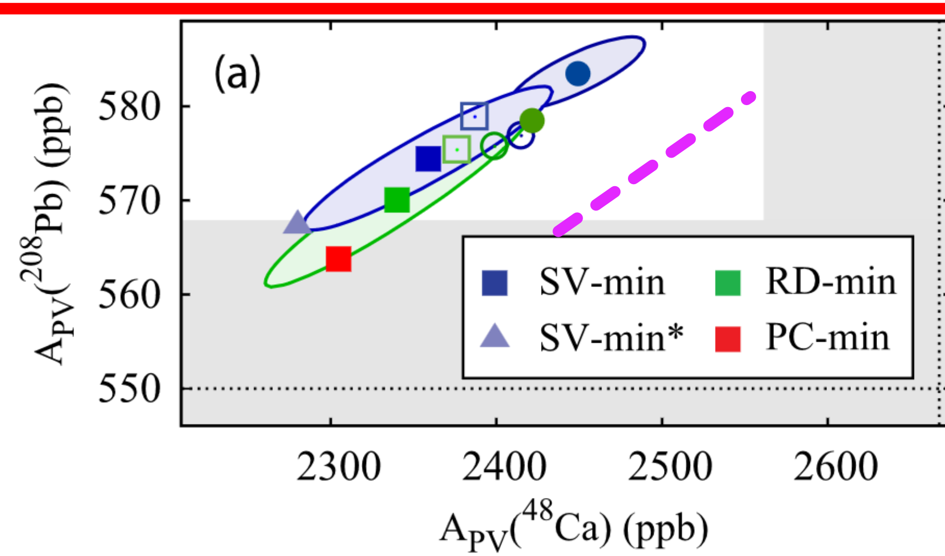
Simultaneous description of **dipole polarizabilities** → point to a **good understanding** of **symmetry energy (J)** and **neutron skins (Δr_{np})**

Ab-initio (B. Hu) Nature Physics (2022)

$\alpha_D(^{48}\mathrm{Ca})$ $2.30^{+0.31}_{-0.26}$

$\alpha_D(^{208}\mathrm{Pb})$ $22.6^{+2.1}_{-1.8}$

No simultaneous description of **parity violating asymmetries** (ground state observable) → point to a **deficient understanding** of **neutron skins**



Magenta dashed lines from extrapolated Δr_{np} given in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Combined Theoretical Analysis of the Parity-Violating Asymmetry for ^{48}Ca and ^{208}Pb

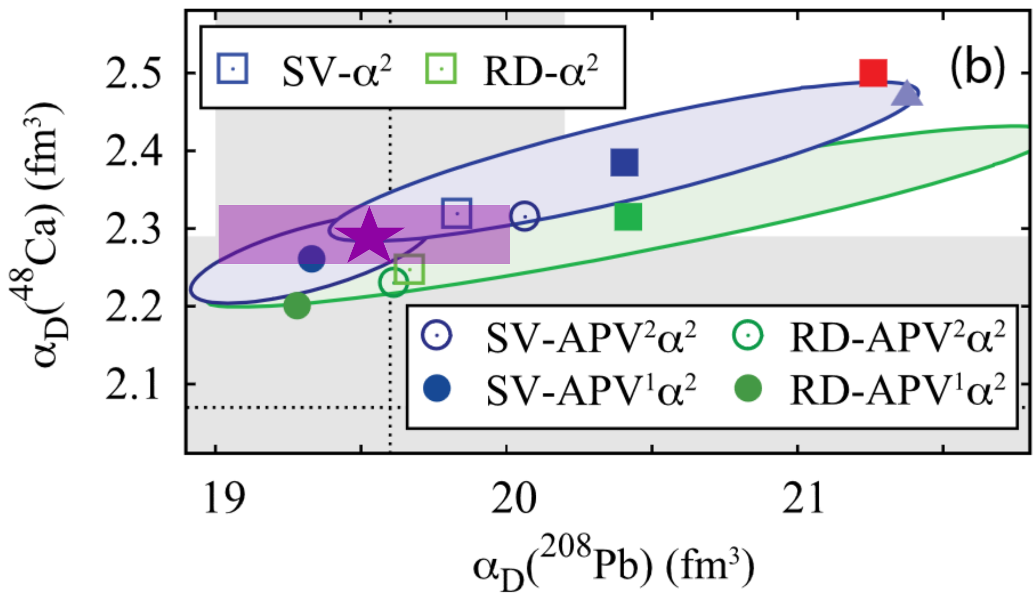
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. **129**, 232501 – Published 2 December 2022

Bayesian inference Skyrme EDF: can we accommodate α_D and A_{PV} without compromising other observables?

Pietro Klausner, Gianluca Colò, Xavier Roca-Maza, and Enrico Vigezzi
Phys. Rev. C 111 014311 (2025)

By P. Klausner

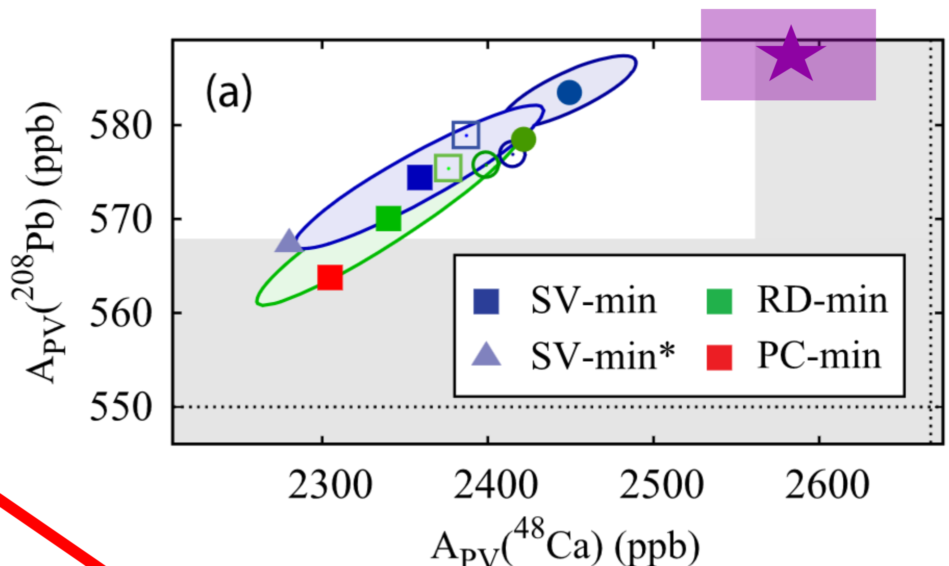
	α_D (fm^3)	$m(1)$ (MeV fm^2)	A_{PV} (ppb)
^{208}Pb	19.5 ± 0.5	958 ± 22	589 ± 5
^{48}Ca	2.30 ± 0.08	–	2591 ± 54



	B.E. (MeV)	R_{ch} (fm)	ΔE_{SO} (MeV)
^{208}Pb	1636 ± 1.8	5.49 ± 0.03	2.34 ± 0.16
^{48}Ca	417 ± 1.2	3.51 ± 0.02	1.92 ± 0.20
^{40}Ca	342 ± 1.6	3.50 ± 0.02	–
^{56}Ni	482 ± 1.4	–	–
^{68}Ni	590 ± 1.0	–	–
^{100}Sn	826 ± 1.6	–	–
^{132}Sn	1103 ± 1.7	4.71 ± 0.03	–
^{90}Zr	784 ± 1.3	4.27 ± 0.02	–

Isoscalar resonances

	$E^{\mathrm{IS}}_{\mathrm{GMR}}$ (MeV)	$E^{\mathrm{IS}}_{\mathrm{GQR}}$ (MeV)
^{208}Pb	13.5 ± 0.3	10.8 ± 0.4
^{90}Zr	17.8 ± 0.4	–

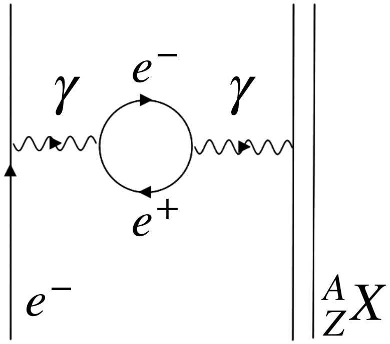


Keeping ground and excited state **properties** within typical **Skyrme-EDF accuracy**

A_{PV} theoretical corrections?

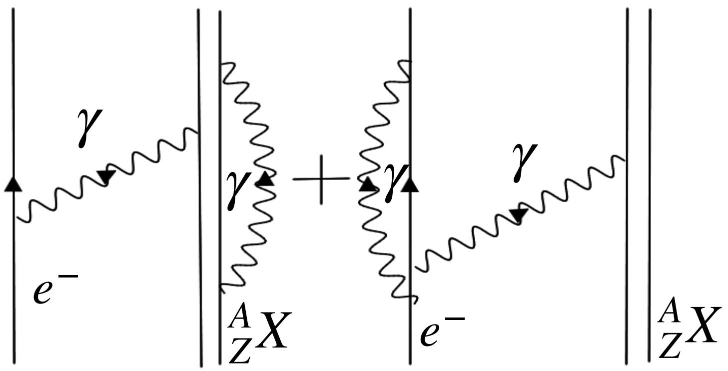
In all analysis: Coulomb potential has been considered at tree level while Q_{Weak} has been corrected at one-loop level (interference of the γ and Z_0 exchange):

$$[-i\vec{\alpha}\cdot\vec{\nabla}+V_{\mathrm{Coul}}\pm V_{\mathrm{Weak}}]\Psi_{\pm}=E\Psi_{\pm}$$



First order QED includes: vacuum polarization (VP), self-energy and e^- vertex correction (V-SE) [Jakubassa-Amundsen (2024) JPG: Nucl. Part. Phys. 51 035105]

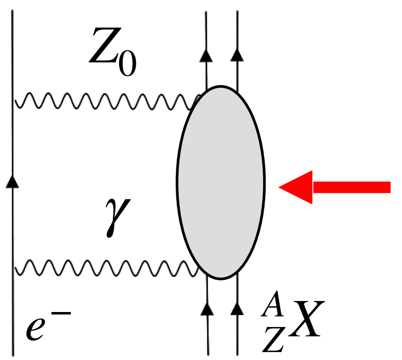
$$[-i\vec{\alpha}\cdot\vec{\nabla}+V_{\mathrm{Coul}}\pm V_{\mathrm{Weak}}+V_{\mathrm{VP}}+V_{\mathrm{V-SE}}]\Psi_{\pm}=E\Psi_{\pm}$$



Dispersion corrections: interference of the γ and Z_0 exchange have been shown to be relevant for the Q_{weak} but effects on F_{weak} [*] have not been studied … [Gorchtein and Horowitz PRL 102, 091806 (2009)]

… but estimated to be **smaller than QED corrections[*]** for the kinematics of PREx and CREx

[*] Jakubassa-Amundsen and XRM, work in progress

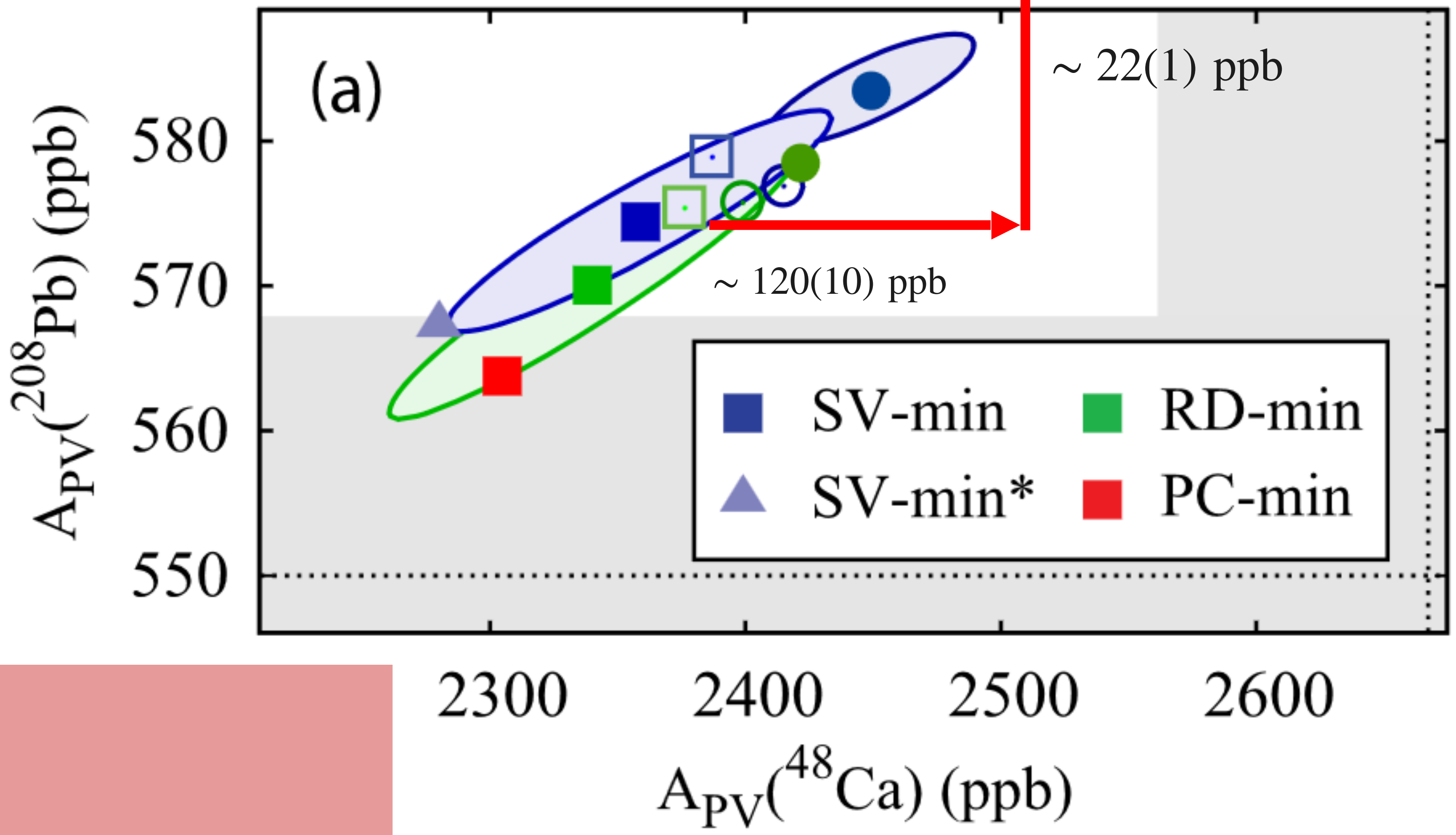


Leading QED corrections:

Jakubassa-Amundsen and XRM arXiv:2501.14375

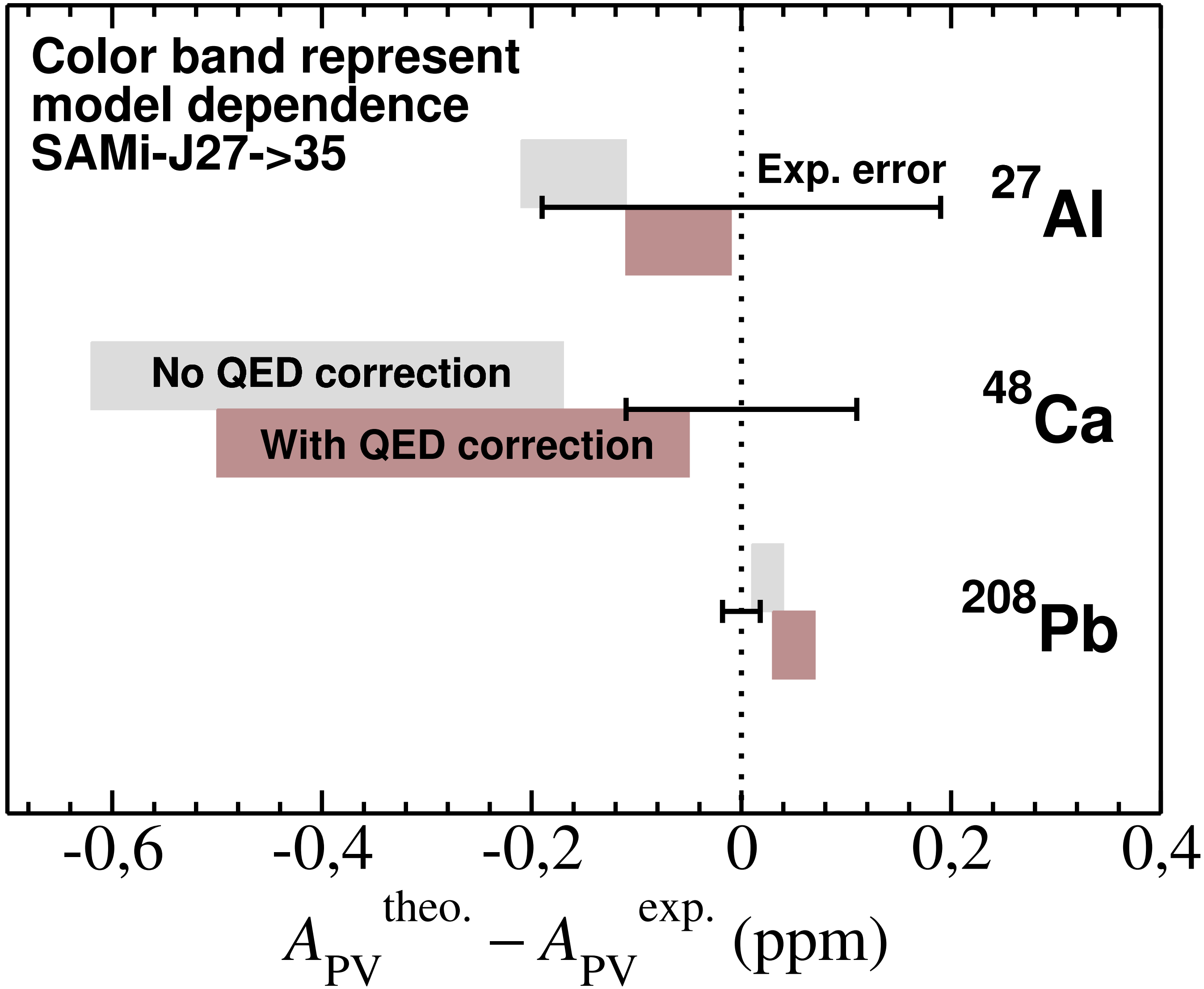
QED corrections (VP+V-SE) are model independent and make A_{PV} larger by about a 5% for the PREx and CREx kinematics

Model dependence of the correction and numerical accuracy within (…)



Leading QED corrections: PRELIMINARY

Jakubassa-Amundsen and XRM, arXiv:2501.14375



How do A_{PV} and α_D reconcile with each other?

Different ways to learn about the **neutron distribution** in atomic nuclei using **different observables** provide **different answers**

However, within **1.5–2 σ all looks fine**, both from phenomenologic **Energy Density Functionals** and from **Ab Initio**

From Theory:

→ An effort to better understand the **parity violating asymmetry** and the **beam normal spin asymmetry** in ^{208}**Pb** is needed [PREX & CREX, PRL 128, 142501 (2022)].

→ An effort to better understand the **systematics** on the **dipole polarizability** is needed [see e.g. Sn chain in Bassauer et al. PLB 810, 135804 (2020)].

From Experiment:

→ An effort to improve the accuracy in the **parity violating asymmetry in** ^{208}**Pb** (and/or measure **other Q values**) and to confirm the measured values for the **beam normal spin asymmetry is needed**.

→ **Systematic** measurements of the **dipole polarizability** along **neutron rich** isotopic chains (e.g. N>74 Sn isotopes)

Collaborators

- → Gianluca **Colò** (University of Milan)
- → Pietro **Klausner** (University of Milan)
- → Hiroyuki **Sagawa** (University of Aizu & RIKEN)
- → Tomoya **Naito** (University of Tokyo & RIKEN)
- → Shihang **Shen** (Forschungszentrum Jülich)
- → Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)
- → Jorge **Piekarewicz** (Florida State University)
- → Nils **Paar** & Dario **Vretenar** (University of Zagreb)
- → Bijay K. **Agrawal** (Saha Institute of Nuclear Physics)
- → P.-G. **Reinhard** (University of Erlangen-Nürnberg)
- → Witold **Nazarewicz** (FRIB and Michigan State University)

Electric Dipole Polarizability: theory

Theoretically, the total **photo-absorption cross section**, can be written as

$$\sigma_{\gamma-\mathrm{abs}} = 4\pi^2\alpha \sum_{\nu}(E_\nu - E_0)|\langle\nu|F_{\mathrm{dipole}}|0\rangle|^2$$

Dipole operator subtract CM motion

$$\frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1M}(\hat{r}_i) - \frac{eZ}{A}\sum_{i=1}^{N} r_i Y_{1M}(\hat{r}_i)$$

And, thus,

$$\alpha_D = 2\sum_{\nu\neq 0}\frac{|\langle\nu|F_{\mathrm{dipole}}|0\rangle|^2}{E_\nu - E_0} \equiv 2m_{-1}$$

Considering the G.S. perturbed by an **external field** λF (with λ → 0):
$\langle\mathscr{H}\rangle = \langle\mathscr{H}_0 + \lambda F_{\mathrm{dipole}}\rangle$; The variation in the expectation energy can be written as:

$$\delta\langle\mathcal{H}\rangle = \lambda^2\sum_{\nu\neq 0}\frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^3) = \lambda^2 m_{-1} + \mathcal{O}(\lambda^3)$$

$$m_{-1} = \frac{1}{2}\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\lambda^2}\bigg|_{\lambda=0}$$

Dielectric Theorem

Electric Dipole Polarizability: simple model & correlations

Applying the **dielectric theorem** to the **Droplet Model** Hamiltonian (first Migdal and latter on Meyer et al. NPA385, 269) one can find

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}$$

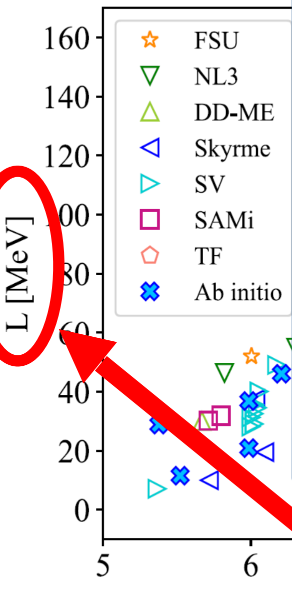
) → Symmetry energy at ρ_0

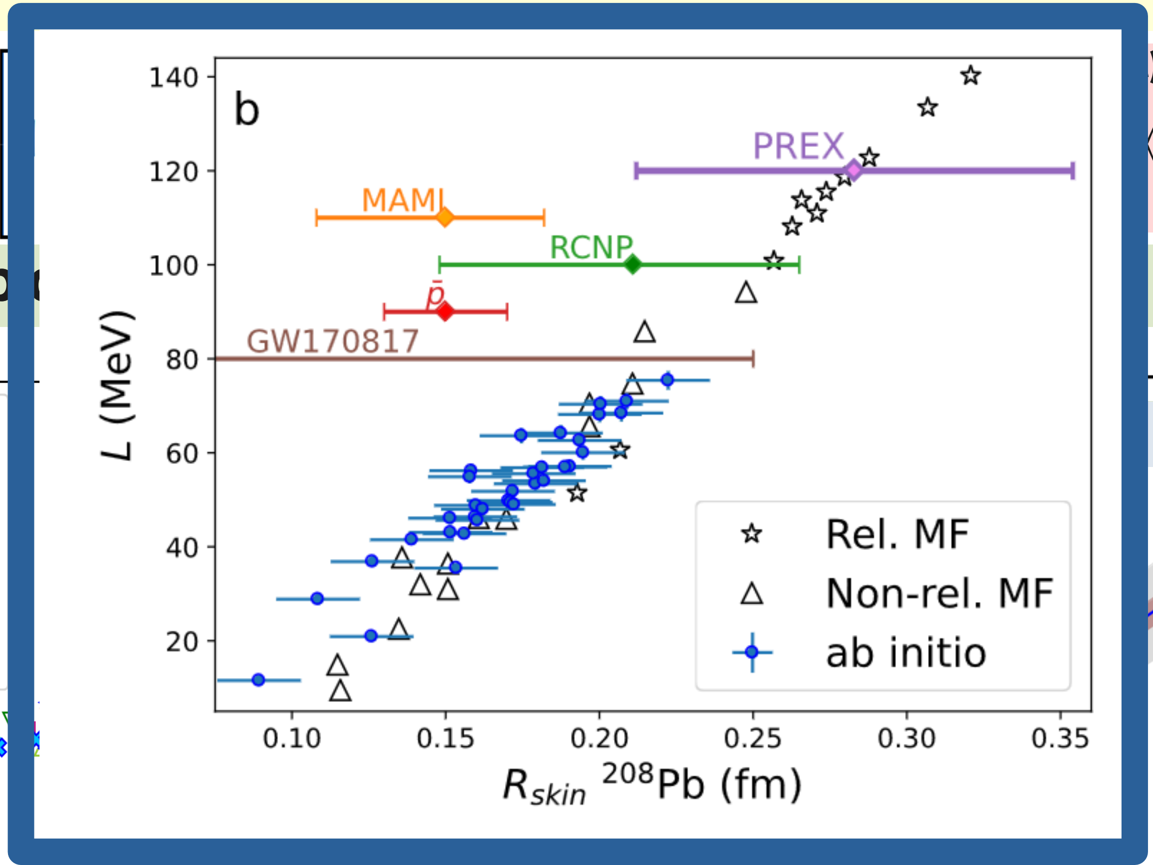
$\langle r_p^2\rangle^{1/2}$ → Neutron skin

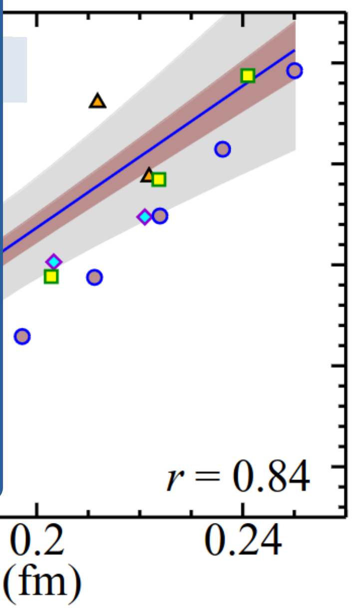
Using microscop

DFT data from: Roca-Maza, Phys. Rev. C **88**, 024316 (2013)

Ab initio data from: Hu, Nat. Phys. **18**, 1196-1200 (2022)



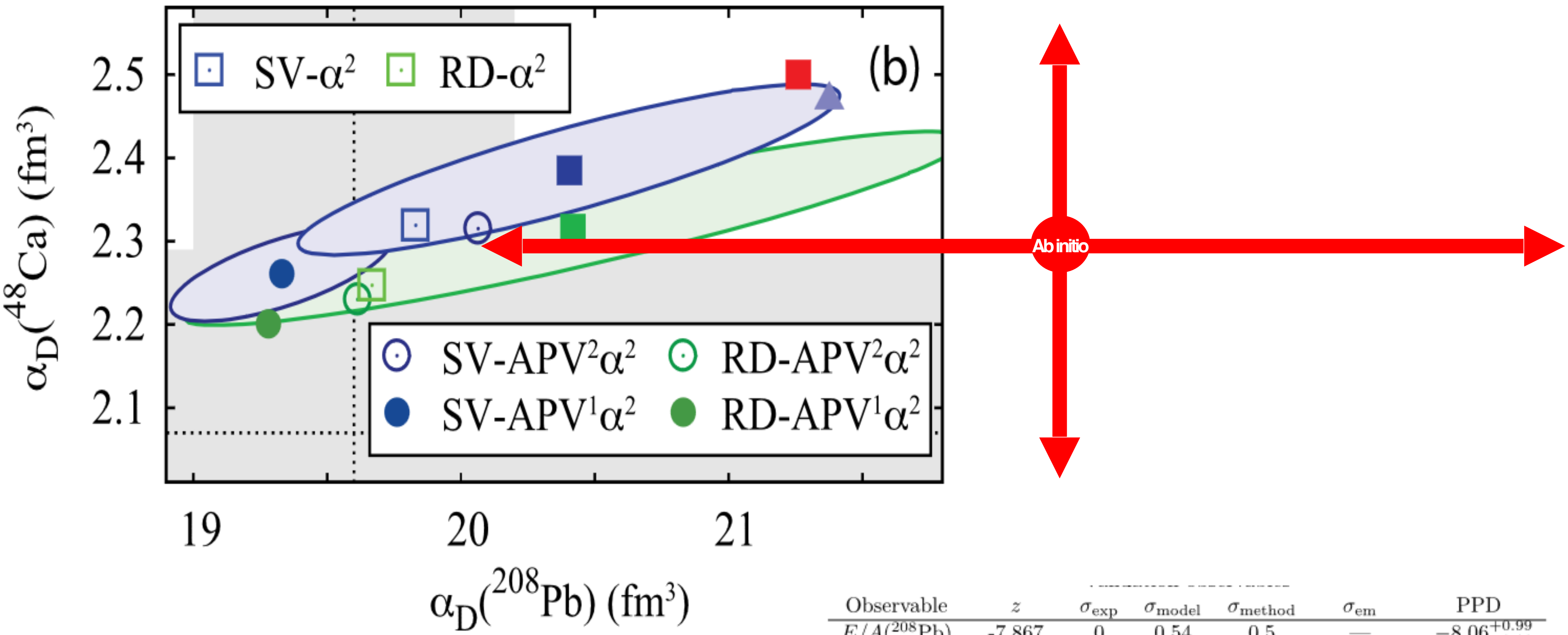




Phys. Rev. C 92, 064304 – Published 8 December 2015

$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

Electric Dipole Polarizability in ^{48}Ca and ^{208}Pb



Observable	z	σ_{exp}	σ_{model}	σ_{method}	σ_{em}	PPD
$E/A(^{208}\mathrm{Pb})$	-7.867	0	0.54	0.5	—	$-8.06^{+0.99}_{-0.88}$
$R_{\mathrm{p}}(^{208}\mathrm{Pb})$	5.45	0	0.17	0.05	—	$5.43^{+0.21}_{-0.23}$
$\alpha_D(^{48}\mathrm{Ca})$	2.07	0.22	0.06	0.1	—	$2.30^{+0.31}_{-0.26}$
$\alpha_D(^{208}\mathrm{Pb})$	20.1	0.6	0.59	0.8	—	$22.6^{+2.1}_{-1.8}$

Combined Theoretical Analysis of the Parity-Violating Asymmetry for ^{48}Ca and ^{208}Pb

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. **129**, 232501 – Published 2 December 2022

Ab initio predictions link the neutron skin of ^{208}Pb to nuclear forces

Baishan Hu, Weiguang Jiang, Takayuki Miyagi, Zhonghao Sun, Andreas Ekström, Christian Forssén, Gaute Hagen, Jason D. Holt, Thomas Papenbrock, S. Ragnar Stroberg & Ian Vernon

Nature Physics **18**, 1196–1200 (2022) | Cite this article

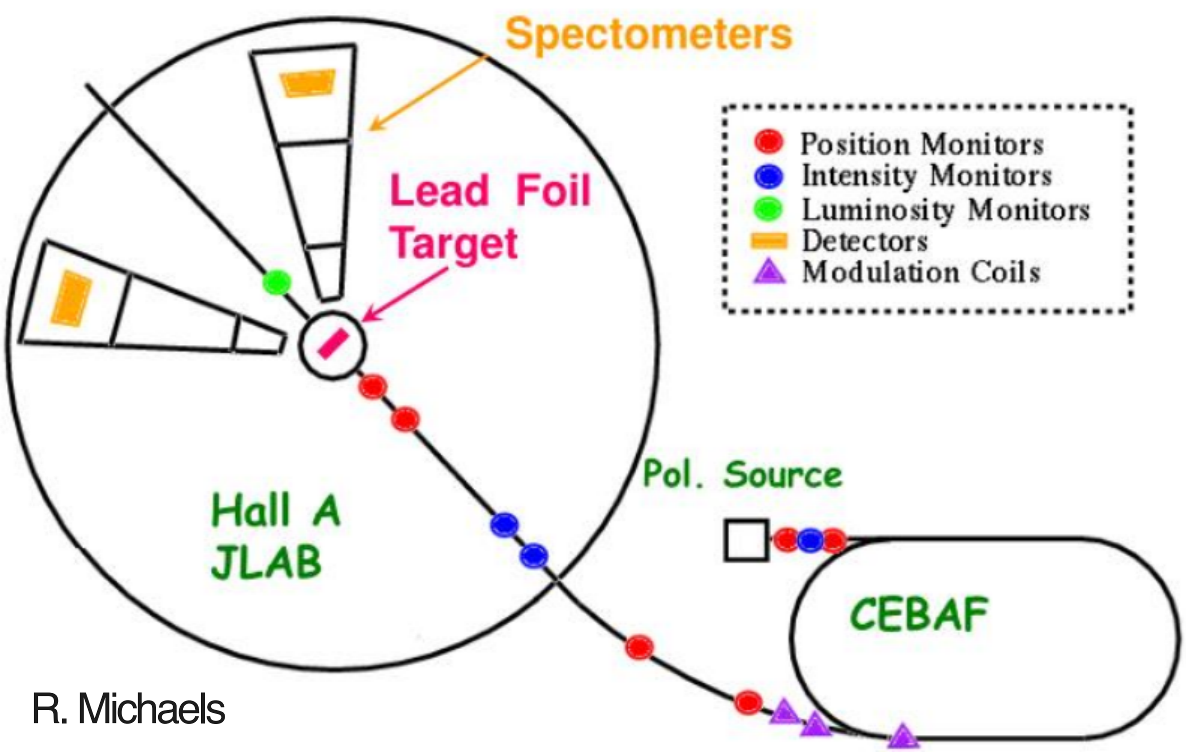
Parity Violating Asymmetry: experiment

Lead and Calcium

Radius Experiments

PREx & CREx @ JLab

$$A_{PV} \sim \frac{G_F}{4\sqrt{2}\pi\alpha} Q^2 \sim 10^{-4}(Q[\mathrm{GeV}])^2$$



R. Michaels

→**Spectrometers ~ 5^0**

→**e^- beam E ~ GeV**

$$Q^2_{\mathrm{PREx}} = 0.00616\ \mathrm{GeV}^2$$

$$Q^2_{\mathrm{CREx}} = 0.0297\ \mathrm{GeV}^2$$

→ **Demanding experiment:**

For **10^6 electrons** or more only **one** of them **interact weakly** with the nucleus

Proposed new experiment on ^{208}Pb @ Mainz !!! (MREX)

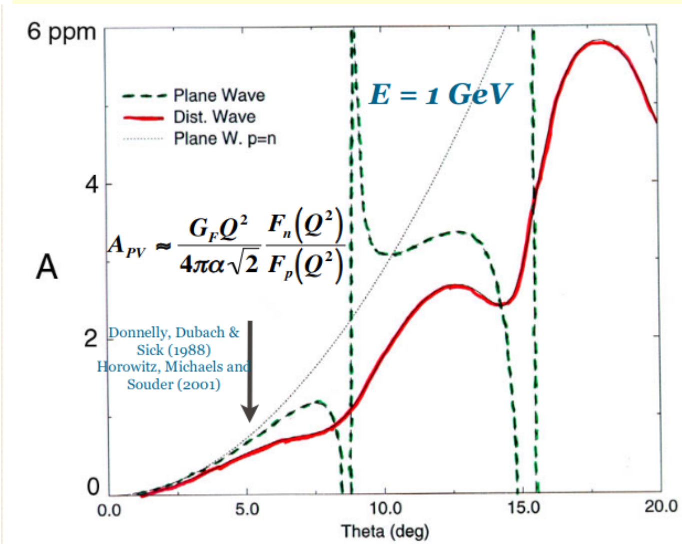
Accurate Determination of the Neutron Skin Thickness of ^{208}Pb through Parity-Violation in Electron Scattering
D. Adhikari *et al.* (PREX Collaboration)
Phys. Rev. Lett. **126**, 172502 – Published 27 April 2021

Precision Determination of the Neutral Weak Form Factor of ^{48}Ca
D. Adhikari *et al.* (CREX Collaboration)
Phys. Rev. Lett. **129**, 042501 – Published 20 July 2022

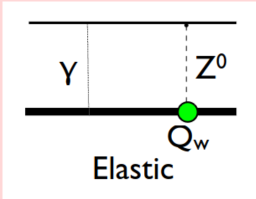
Parity Violating Asimmetry: theory

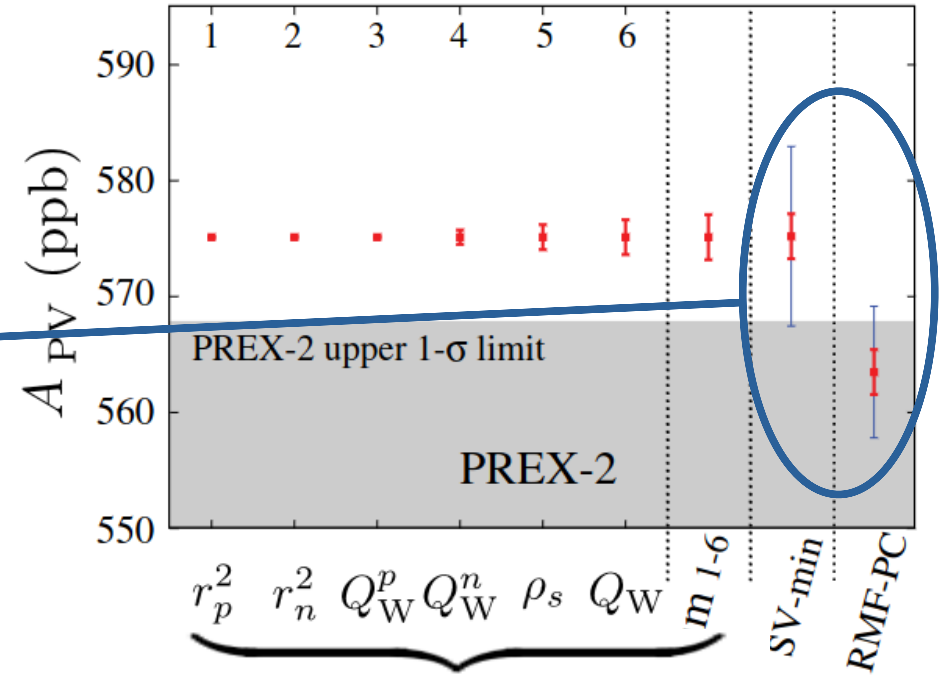
The **main uncertainties** come from **nuclear model errors** on the description of the **neutron distribution (blue error bars)**

→ **Coulomb distortions are important** (order αZ)



Are **γZ$_0$ box** corrections **important**? not included





Hadronic uncertainties less relevant

Parameter	Value
$\langle r_p^2 \rangle$ (fm^2)	0.726 ± 0.019
$\langle r_n^2 \rangle$ (fm^2)	-0.1161 ± 0.0022
μ_p	2.792 847
μ_n	-1.9130
$Q_p^{(W)}$	0.0713 ± 0.0001
$Q_n^{(W)}$	-0.9888 ± 0.0011
ρ_s	-0.24 ± 0.70
κ_s	-0.017 ± 0.004
$Q_{126,82}^{(W)}$	-117.9 ± 0.3

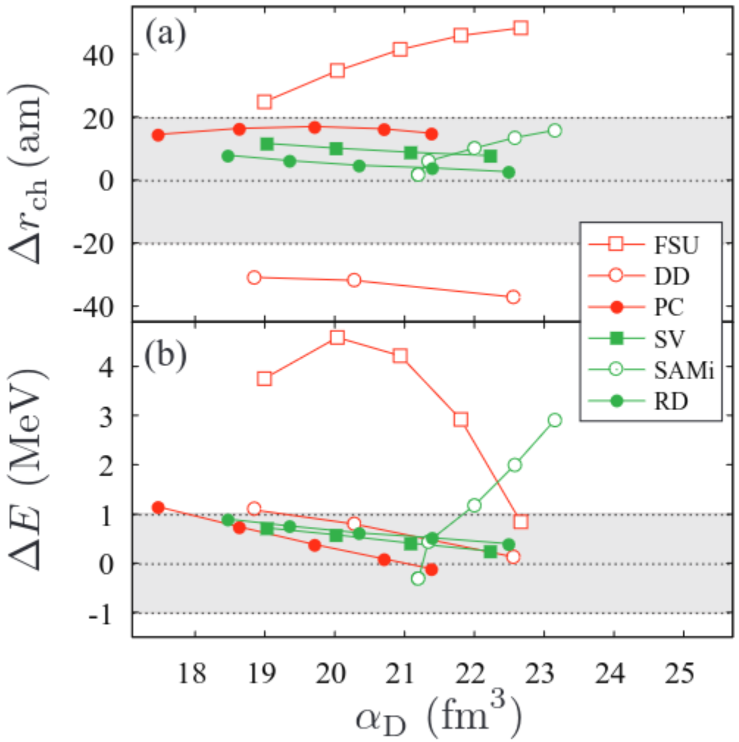
Information Content of the Parity-Violating Asymmetry in ^{208}Pb

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
Phys. Rev. Lett. **127**, 232501 – Published 29 November 2021

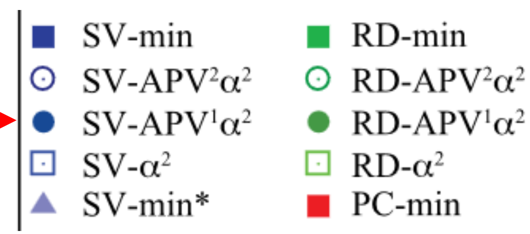
Parity Violating Asymmetry: measurements vs. theory

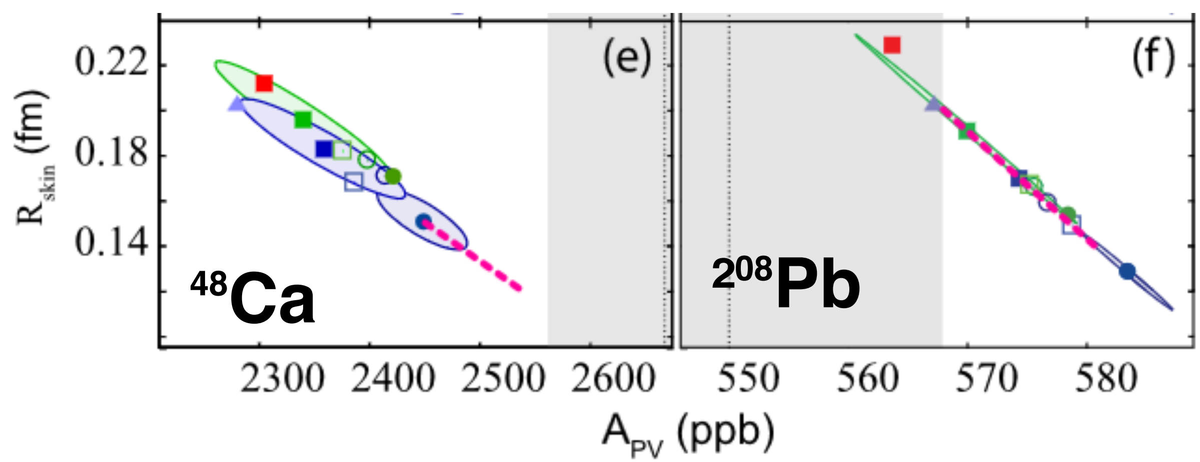
Selecting those **models** (we consider) **well calibrated** to **masses** and **radii**

Theoretical (**EDFs** and **ab initio**) and experimental 1σ errors overlap in ^{208}Pb but not in ^{48}Ca → **No simultaneous description**



Fitting protocol also including **A**$_{\mathbf{PV}}$ and **α**$_{\mathbf{D}}$





Combined Theoretical Analysis of the Parity-Violating Asymmetry for ^{48}Ca and ^{208}Pb

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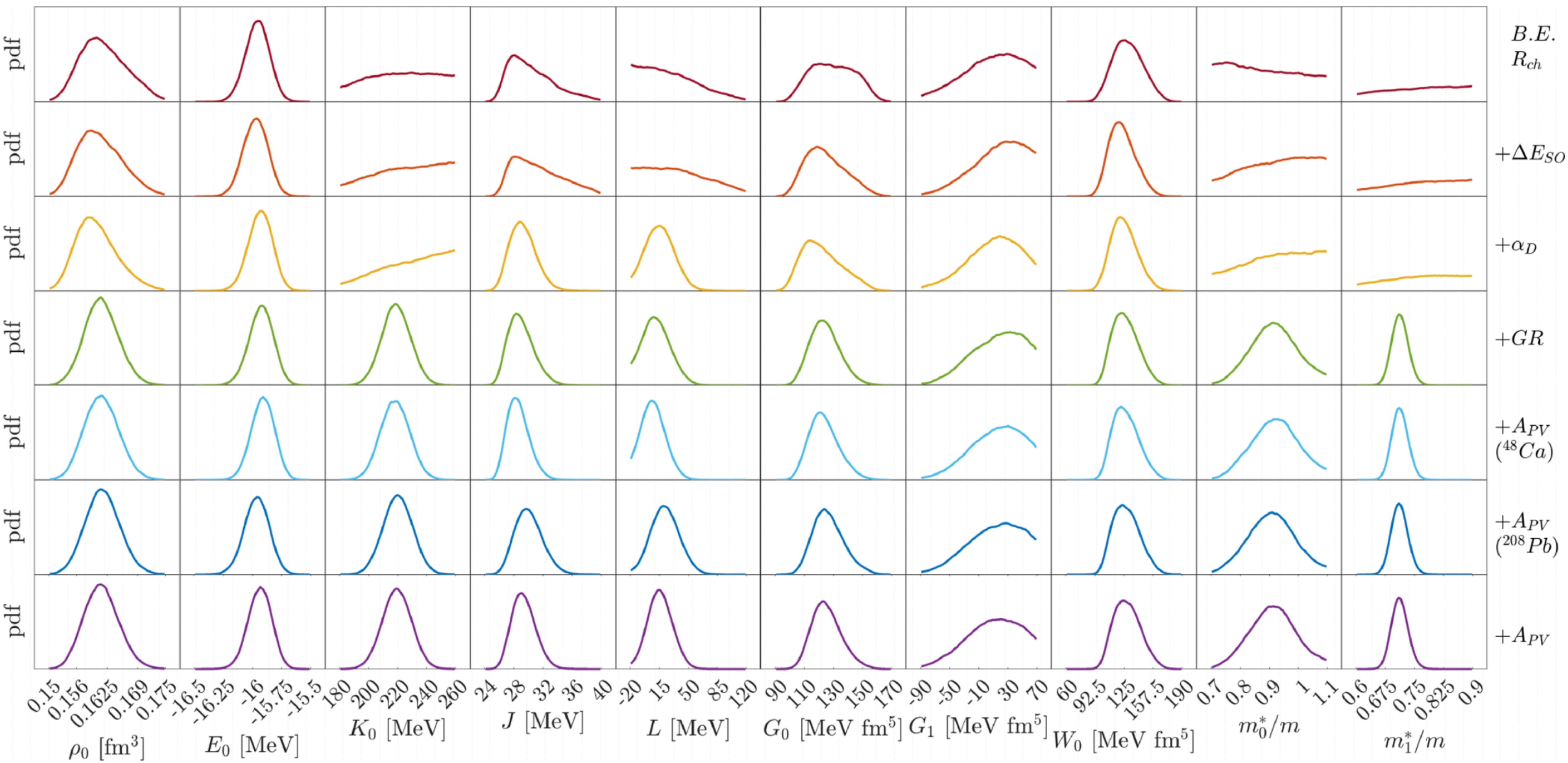
Magenta dashed lines from extrapolated $\Delta r_{\mathbf{np}}$ given in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Bayesian inference Skyrme EDF: can we accommodate α_D and A_{PV} without compromising other observables?

By P. Klausner



Pietro Klausner, Gianluca Colò, Xavier Roca-Maza, and Enrico Vigezzi
Phys. Rev. C 111 014311 (2025)